MATHEMATICAL ANALYSIS

OF SOLID WASTE COLLECTION

This final report (SW-5rg) on work performed under Research Grant No. EC-00309 (formerly UI-00828) to the Johns Hopkins University was written by DAVID H. MARKS and JON C. LIEBMAN Department of Geography and Environmental Engineering and has been reproduced as received from the grantee.

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Public Health Service
Environmental Health Service
Bureau of Solid Waste Management
1970

to aid in developing economic and efficient solid waste managemen practices. As authorized under the Solid Waste Disposal Act (Pub Law 89-272), the Bureau has made almost 100 research grants to no institutions in this effort to stimulate and accelerate the devel of new or improved ways for handling the Nation's discarded solid present document, which is reproduced as received from the grante reports on work completed under one of these research grants.

The general problems were investigated by the grantee in an lytical framework. These are the location of transfer stations a routing of collection vehicles. Mathematical optimization models these problems were constructed, using existing or slightly modif algorithms. These models, particularly the ones for transfer fac site selection, may be used in choosing the economically optimal from among a large number of alternatives. This information serv nonquantifiable information in making the final choice.

Several of the models were teated by the grantee using data from the City of Baltimore. Feasibility of transfer stations, th sirability of rail haul, and the cost of increased collection fre were investigated in the testing process. These applications ser demonstrate the final usefulnesa of the models.

--RICHARD D. VAUGHAN, *Directo*
Bureau of Solid Waste Manag

ABSTRACT

The application of operations research to the a
solid waste collection systems is studied. Models anc
for facility location and routing are discussed and ex
Three different types of problem areas are viewed. Cl
covers facility location, in particular the location c
facilities within a large-scale system. Chapter III c
finding optimal flow through given systems with added
In particular, a multi-commodity truck assignment prol
a common vehicle fleet is used to carry several commod
between supply and demand points is extended and solve
Chapter IV is concerned with vehicle scheduling proble
routes are to be found for individual collection vehic
various tasks. In Chapter V, the analysis of an actua
scale solid waste collection system, that of Baltimore
is carried out using some of the methods developed. I
the analysis indicate that the use of such models in t
of large-scale public systems can supply a great deal
information about, and insight into, the management ar
of the system.

LIST OF TABLES

LIST OF FIGURES

CHAPTER I. OPERATIONS RESEARCH AND SOLID WASTE COLLECTION

roduction

This thesis is concerned with the development of analytical hniques to aid in the management and planning of solid waste lection systems. Specifically, the techniques to be presented operations research models of routing and facility location blems that are an inherent part of any collection or delivery tem. The models are most applicable in the private sector of the nomy, since they seek to minimize a well-defined and quantifiable t subject to explicit constraints. Solid waste collection tems, however, are for the most part functions of the public tor rather than the private sector. There is a divergence be- en the objectives of a public and private system, with the public ective function being more vague and difficult to express formal- Constraints on the public system, especially those of a itical or social nature, may be difficult to measure, and criteria effectiveness may not exist in units which are commensurate.

All this does not mean that operations research models and ls should not be applied to public sector problems such as solid te collection system management. As with private sector models, lysis is performed as an aid to the decision maker and not as a lacement for him. Careful use, based on an appreciation of the tors that the models cannot consider explicitly, will allow ningful analysis to be carried out. The amount of analysis

attempted in such large scale systems until recently has been limited by the sheer size and complexity of the problems. Now wi the availability of operations research techniques and the aid o high speed computers, the potential to increase greatly the under standing of such systems is apparent. It is hoped that this will be an essay both in the advantages to be gained by systematic investigation and the pitfalls involved in blindly following mode as a substitute for judgment and intuition.

The purpose of this work is twofold. First, the routing an facility location problems of a collection system will be analyz and specific techniques suggested for approaching their solution Second, the application of these techniques to the investigation a large scale solid waste collection system as an indication of questions amenable to analysis by these models will be presented.

A. The Scope of the Solid Waste Problem

The solid waste system is defined to include the generation waste at a source, its collection and transport and the disposal methods available. It is a large, expensive and vital on-going system with peculiarities and problems that are in urgent need o analysis. The yearly cost of operating the system in the United States is reported by Black (1964) to be 3 billion dollars per y This amount makes it one of the foremost public works expenditur ranking after education, highways, welfare, and fire and police among local government expenditurea. The amount of waste genera

the United States from all sources approaches 125 million tons
r year. With increasing indications that these costs and volumes
ll continue to rise drastically (Ludwig and Black, 1968), it seems
at detailed analysis of the system would be in great demsnd.

The main difficulty in analysis is determining some measure of
fectiveness for the service provided. The evolution through the
ars of solid waste management from a private to a public problem
indicative that the service provided is a public good. The
tire community suffers from the viewpoint of public health and
sthetics when even one of its members refuses to dispose properly
his solid wastes. The cost of enforcing sanitation ordinances
thin a private system msy far outstrip the cost of a publicly
ned system. A measure of efficiency of a public system might be
e degree of service offered to the customer in terms of frequency
collection, types of wastes removed, locations from which waste
collected, and the general level of satisfaction shown by the
nsumers in order to provide the incentive for all to dispose
operly of waste material.

Management to achieve some desired level of service at minimum
st is the goal, then, of the system operator, and he must study
e various control alternatives available to improve the efficiency
the system. Cost analyses by Ludwig and Black (1968) reveal that
percent of the solid waste system cost is due to collection and
ly 15 percent to disposal. This does not necessarily mean that
e only approach to the problem is through improvements in

collection. In spite of the fact that most analysts consider tha waste loads are given and begin designing a system from that poi it would seem that collection costs could be decreased by elimina wastes before they require collection and transport. From a tech logic point of view, materials which are difficult or expensive t handle or dispose of might be replaced by those which are not. T impetus for such a move could be either legislative, through laws preventing the use of objectionable material, or economic, throug special taxes which would make using the materisl economically infeasible. Economic incentives might also change current waste material into production inputs. Education of the public might a reduce waste load produced. Technologic advance in the form of better in-home disposal units is a hope for the future even thoug present devices are less than adequate. Present day incinerators create an air pollution hazsrd and still have an ash disposal problem. Garbage grinders simply trade a solid waste collection system for an underground liquid waste system, and disposal probl continue.

Some ideas have already been advanced for improving collecti Prominent among these is the building of transfer facilities with a city at which the special purpose collection vehicles could dis charge their loads to special purpose transport vehicles and thus return to their collection jobs sooner. The suggested transport vehicles have ranged from large tractor trailers to barges and ra road cars. Recommendations have also been heard for even more

ialized collection vehicles such as small carts hauled in tandem jeep or motor scooter to a rendezvous with a larger transport cle. Zandi (1968) has proposed an extensive pipe line aystem moving solid wastes under pressure aa an alternative to surface ection. All of these control methods require some amount of stigation and testing. Yet, the most important thing to realize t the solid waste system is that it is too big, complex, and l to allow actual experimentation without great expense or the ntial of great chaos. Coupled with this are all the other lems involved with studying large scale public systems. A rele- data baae is probably non-existent. The political implications ho controls the system and who pays for which service may be the persuasive argument for throwing out a scheme that might rwise seem quite efficient. Since large investment has already made in attempting to manage the system, the designer is denied luxury of starting from the beginning and is often aaddled with ing around the blunders of the past. To this morass the yst hopes to bring some order.

Since collection makes up 85 percent of the cost of the solid e management system, it is to this phase that the attention of work will be directed. Successful analysis of the collection ess could result not only in means for making it run more ciently, but also provide a means for measuring how non- ection alternatives which reduce waste load must be priced in r to make them better choices than the current approach to the lem.

B. Literature Review of Operations Research in Solid Waste Col

A search of the pertinent literature reveals that very few authors have concerned themselves with the application of opera research to the study of solid waste collection systems. The w that does appear falls into two categories: broad scale attemp outlining the nature and interactions of the entire waste manag system, in which collection is but one part, and narrower, more specific investigations of the collection and disposal operatio suited to the detailed study of present operating systems. The complete of the broad scale studies is one still under way at t University of California at Berkeley entitled "Comprehensive St of Solid Waste Management". In the first annual report of the (Golueke and McGauhey, 1967) one of the stated objectives of th study is "to explore the potential of operations research to he in the definition and solution of the solid waste disposal prob Basically, two problems were investigated. The first was to fi means for evaluating solutions for the total refuse disposal pr that would not only be applicable to a wide spectrum of alterna but would be sensitive to environmental and governmental constr as well. This involved the development of conceptual models wh gave insight into the operation of the system. As a second pro the study undertook to build mathematical models for the concep models. These included a waste generation model, a waste colle treatment and disposal model, a regional economic model, and mo

for population, public health aspects, land use and process te logy. The most interesting of these models from the viewpoint operations research techniques was the waste collection, treat and disposal model. The subgroup working with this problem lo at the means of determining how solid wastes should move from sources through treatment and processing to disposal sinks. T identified the problem as one of network analysis and noted th solution should be gained by a graph theoretic approach. The come of the development of specific techniques for solving the as formulated is reported by Anderson (1968), who was one of t study group. Anderson was interested in modeling how the flow be optimized through a given system which contained existing facilities and potential new facilities. He identified the pr as a study of a trans-shipment network with some peculiarities to the nature of the processes involved. In considering a tot waste management model including intermediate treatment facili it is necessary to consider the changes in volumes and in the of the wastes which is not possible using ordinary network alg He suggests two solution techniques: if the number of disposa is small, a branch and bound model may be used to generate opt flow through the system. If not, he also presents an out-of-k algorithm for solution which includes an added feature of allo flow through a node to be proportioned in a fixed ratio. In b models all costs are linear, and the system under investigatio considers the gross shipment between points and not the routin

individual vehicles among small collection tasks. Also, dealt with the problem of finding flows through a known configuration. To consider various combinations of new each scheme must be evaluated separately. No methodolog presented for a systematic search over facility location ties. The assumption of linear costs omits some of the aspects of the problem. Although nonlinear costs may be linear approximations, this is only valid if the nonline are convex. Facilities costs are commonly concave or qu (fixed charge). Thus linear approximation either will n successful or must be done in such a manner that one los confidence in the meaning of the results of the solution

Other studies defining the solid waste system have out by consulting organizations for governmental units, any have gone beyond the characterization of the system at simulation of the process. Such studies include repo Management Technology, Inc. (Anon. 1966b) for the State o and by Aerojet General (Anon. 1965) for the State of Cal

The first work in attempting to understand the comp microstructure of the detailed collection process at the block and vehicle level has been by simulation models. and Wersan (1965) presented a model of a collection syst particular attention to queuing problems at the disposal using data from Winnetka, Illinois. Truitt, Liebman and 1969) constructed a simulation model in which the effect

proposed transfer station may be investigated. Their results

based on data from Baltimore, Maryland. Quon, Tanaka and Wer

(1969) continue their earlier simulation work with a model fo

studying changes in work rules and collection policy.

Other authors have dealt with different aspects of the c

tion problem that are more amenable to analytical attack. Co

and Martin (1967) presented a heuristic method based on dynam

programming for aggregating small collection areas into work

schedule assignments for crews and vehicles. Skelly (1968) p

ted a fixed charge model for looking at the large scale probl

transportation of wastes in which the variables include the a

native location of transfer stations. He used for a solution

technique a heuristic fixed charge algorithm developed by Wal

(1968). Skelly's work is discussed in greater detail in Chap

Wolfe and Zinn (1967) have presented a simple model for an ov

crude evaluation of a large system as an example of systems a

cation in public works problems. A small trial problem was s

using linear programming.

C. The Analysis of Solid Waste Collection

The analyst involved with a solid waste collection syste

concerned with answering a set of questions about the system

he can better understand it. Thia understanding will lead to

improved decision making. The sort of questions he might ask

the following:

1. What are the goals of the system? What freque
collection and types of service should be offer
system? How will changing the service affect c
2. What types of vehicles should be used, and how
3. How many personnel are needed and what should t
duties and work rules be?
4. What route should be assigned to each vehicle?
the city be divided into administrative subgrou
5. Are there parameters of the system to which syst
snd variables are particularly sensitive?
6. If there is additional money available for resea
what aspect of the system should further study b
7. Should there be intermediate transfer stations fo
deployment of wastes to more specialized transpor
Where should they be located and what type of equ
should they contain?
8. What type of transport vehicle would be used in t
from a transfer station to final disposal?
9. What type of disposal alternative should be choaen
where should it be located?
10. What would be the effect on the system of new tech
in in-house waste reduction? In new disposal tech
11. How will the stochastic nature of waste generation
the analysis? How will the solution change as the
be served continues to grow and spread?

12. What are the effects of political, social and economic constraints? How much should be spent on aesthetic factors? Is regional grouping a feasible slternative?

Ideally, to answer all these questions, the analyst would l to build a model that would be able to follow the collection pro in its minutest detail and be able to manipulate all of the myri parameters that could possibly affect the solution. However, it magnitude and complexity are such that to consider the system in entirety and encompass every possible detail is quite difficult, not impossible. For this resson, simplifying assumptions will b made in model development which do not allow the detail mentione but will allow some approximation of the problem. Basically, tw different approaches are tsken. If one is willing to neglect th problems of routing of the individual vehicles through their col tion tasks, the large scale problem of how material should move through the system and how transfer facility location may be cho can be approached optimally. This will be called the "macro-sca or "flow of materials" subproblem. If the location of facilitie and the collection areas assigned are known for a set of collect vehicles, the small scale problems of routing vehicles among individual collection tasks may be approached. Thia second sub-problem will be called the "micro-scale" problem. Extensive discussion and development of both of these problems will be pre-sented. Application of the proposed models to the investigation a solid waste collection system will be emphasized.

The general outline of this thesis is as follows: [cut off]
is concerned with the location of transfer facilities. [cut off]
problem is treated as a large scale "flow of materials" [cut off]
is found to belong to a class of problems known in the li[cut off]
as location problems, warehousing problems and plant loca[cut off]
problems. Techniques for solving these problems are disc[cut off]
a special method is developed that is particularly adapta[cut off]
solid waste collection problem.

Chapter III also deals with the large scale flow [cut off]
problem. In this case, the network structure of the syst[cut off]
and optimal flow through it is found for more extensive [cut off]
than were allowed in Chapter II. In particular, multicom[cut off]
using a common vehicle fleet is studied. Extension of a [cut off]
modity truck assignment problem by Szwarc (1967) is prese[cut off]
computational experience is discussed.

Chapter IV deals with the micro-scale routing prob[cut off]
are found to be represented by several classes of problem[cut off]
tions research. The most general problems discussed are [cut off]
vehicle scheduling problems and traveling salesman proble[cut off]
original algorithm for the m-salesman traveling salesman [cut off]
presented. Problems of routing in street networks are sh[cut off]
cases to be chinese postman problems, and this problem is[cut off]

Chapter V deals with the analysis of an actual sys[cut off]
of the techniques developed in the thesis with fairly cru[cut off]

Chapter VI presents a summary, and some conclusion[cut off]
suggestions for further research.

CHAPTER II. THE LOCATION OF TRANSFER FACILITIES

A. Problem Description and Literature Review

Determining the location of intermediate facilities where transfer of solid waste may take place from specialized collec vehicles to vehicles more suited for long-haul transportation problem that has received considerable attention within the pa years. The fundamental questions involve the desirability of transfer stations, their number, location, and capacity, and th specific functions which they should perform. These decisions be viewed as a trade-off between the building of facilities and cost of transportation. Two points are immediately clear. Fi this is a general problem of facilities location in both the pr and public sector rather than a problem specific to solid waste and studies in the general area of facilities planning may be applicable. Second, assuming that facility costs and transport costs can be defined, it may be possible to derive some mathema formulation of the problem.

A search of the operations research and economics literatu reveals that a considerable interest has been generated and mat matical formulation for certain aspects of the problem has been developed. The methods developed have been for the large scale "flow of goods" features of the problem in which the concern ha been with selecting paths over which materials flow from collec areas to facilities for transfer and disposal. The routing of

icles within collection areas is usually ignored, as it would a complexity to the problem that would make it intractable. ever, since the goal of model formulation is to obtain some ucture that may be solved and manipulated in order to gain better erstanding of the system as a whole, such work is still of great ortance. Care should be taken, though, in the use of the solu-ns generated. An optimal answer to a subproblem may have little ning in the context of the larger problem, even though manipula-n of the subproblem to show the sensitivity of its optimal ution to changes in parameters and control methods may provide reat deal of insight and information for the analyst.

The problem area as discussed in the general literature is wn by several names: the location-allocation problem, the plant ation problem, the central facilities problem and the warehousing blem. To a large extent, all of these names imply the same eral problems, with differences depending on the assumptions made order to facilitate modeling and solution techniques for the els. As noted, the general formulation is based on trading off lity cost versus transportation coat. The problem is to find t system configuration that minimizes the sum of the two costs.

Analysts who have worked on this problem have been faced with basic problems in definition and assumptions which have led to different structures for analytical approach. The first assump-n is whether demand is spread, uniformly or otherwise, continuous-across the entire area to be served, or is located at discrete

ints. The second assumption is whether potential facilities sites e infinite in number or are limited to a discrete number of asible alternatives. If the assumption is made that both demand d facilities locations are infinite the analysis procedure is ite ill-defined and little analytical success has been obtained such problem solutions. However, once some form of limiting sumption is made of the solution space, considerable progress in lution technique has been found. It should be pointed out that ch discrete assumptions are not severe from an engineering point view. In fact, within a city there are not an infinite number sites for solid waste facilities, but a finite number based on ailable land, zoning and concentrations of population and the cation of existing structures. Demand, too, does tend to cluster d an assumption dividing a region into areas with each represented its demand at some centroid is not unreasonable in most cases. e two approaches may be structurally distinguished by regarding e problem with infinite solution space for demand and facility cation as location on a plane, and the discrete problem as cation on a network.

The problem names most associated with location on a network e the warehousing problem and the plant location problem. In the rehousing problem, warohouses or central facilities are built to rve as supply points for satiafying demand in the region and the ow of goods takes place from the facility to the demand area. In e plant location problem, the central facility is a production cility which requires a flow of goods to it from the surrounding

regions. From an analysis point of view, the problems and the problem names may be used interchangeably. Th description of a warehouse problem may be converted to of the plant location problem by changing the directio goods.

The statement of the warehouse problem is: Given demand areas for a certain product, each with a demand number of alternative sites where facilities may be bu these demands, determine where facilities should be bu demand areas are to be served by each facility. The o that the sum of the transportation cost and the amorti cost is minimized.

The general mathematical formulation ia:

$$\text{Minimize:} \quad \sum_{j=1}^{n} \sum_{i=1}^{m} d_{ij}(x_{ij}) + \sum_{i=1}^{m} F_i(y_i)$$

$$\text{subject to:} \quad \sum_{j=1}^{n} x_{ij} = y_i \qquad i=1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad j=1, 2, \ldots, n$$

$$x_{ij} \geqslant 0 \qquad i=1, 2, \ldots, m; \; j=1, 2, \ldots, n$$

$$y_i \geqslant 0 \qquad i=1, 2, \ldots, m$$

where: x_{ij} = amount shipped from warehouse demand area j (a decision vari

y_i = total amount shipped from warehouse i (a decision variable)

$d_{ij}(x_{ij})$ = cost of shipping x_{ij} from i to j (dolla

$F_i(y_i)$ = coat of building and operating warehous when y_i is to be shipped from it (dolla

D_j = demand at area j

n = number of demand areas

m = number of proposed warehouse sites.

If the functions $F_i(y_i)$ and $d_{ij}(x_{ij})$ are linear, the proble mes a simple tranaportation problem. Unfortunately, the funct $_i$) is frequently nonlinear, and generally exhibits a large fix stment for land, foundations, utilities, etc., before any amou be manufactured or shipped. Once the facility is begun, the inal cost of another unit of storage or manufacture may decrea nomies of scale). Such a function is termed concave and is no able to linear programming. This is the feature which has dra most attention from researchers in this field, and the method ting the concave cost function is the key element of most edures.

Among the earliest work in the literature is a paper by Bau Wolfe (1958), in which the authors approach the problem by usi near facility cost instead of a fixed charge. Then the proble be formulated as a transportation problem. The resulting solu is then adjusted, using a heuristic procedure, to reflect the d charge aspects of the problem. This discussion will be ted to five fairly recent approaches, all thought to have some

application and validity. The papers by Feldman, Lehrer and Ray (1966) and by Kuehn and Hamburger (1963) represent two early formulations of the problem which rely on heuristics to achieve good solutions. Balinski (1965) formulated the problem as an integer programming problem which is capable of solution for small cases. Efroymson and Ray (1966), Spielberg (1969) and the author of this thesis propose methods which achieve optimal solutions to larger mathematical problems as structured through branch and bound techniques.

As noted before, Kuehn and Hamburger used a heuristic algorithm to find good solutions to the warehousing problem. They assumed that transportation costs are linear functions of the amount transported and that facility costs are of the form,

$$F_i(y_i) = a_i + b_i y_i \quad \text{if the facility exists}$$
$$= 0 \quad \text{if it does not}$$

Thus $F_i(y_i)$ consists of a fixed charge that is independent of the storage and a linear cost which does depend on storage, if the facility exists.

The method starts with one facility and tries adding another facility to aee if the total cost can be decreased. The general assaumption is that the best N facilities contain the set of the best N-1 facilities. Tarmination of the procedure occurs when it appears that another facility cannot be added without increasing the total cost.

One difficulty in working with heuristics is that there is n

to tell how far the final anawer obtained is from the true
:imal solution. This difficulty tends to decrease the validity
an analysis of senaitivity by obscuring the cause of a change in
aolution which might occur when a parameter is altered. An
provement could be due to the parameter change or to luck in the
nputations. Thus, extreme care ahould be taken in using a
iristic for aensitivity analysis.

Feldman, Lehrer and Ray assumed a more general form for the
:ility coat, that of a continuous concave function. They also
sumed that tranaport cost is linear in the amount shipped between
nts. Since $F_i(y_i)$ does not have a linear variable cost but s
riable cost that is dependent on how many demand areas sre being
rved by the facility, the problem of assigning demand areas to
sting facilities is not as easy aa before. An approximation
thod is uaed to accompliah thia asaignment. Whereas Kuehn and
nburger began with one facility, Feldman et al. started with all
tential facilities exiating and dropped some out. Termination
curs when no further savings can be obtained by dropping facili-
ea. Balinski (1964, 1965) developed a formulation for the plant
cation problem that could be aolved by integer programming tech-
quea. In order to do this, $F_i(y_i)$ ia apecified as a single fixed
arge, F_i, if the facility exista. Capacity constrainta are not
ecified for the facilitiea. The formulation is as followa:

nimize: $$\sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij} x_{ij} + \sum_{i=1}^{m} F_i y_i \qquad (2\text{-}6)$$

subject to: $\sum_{i=1}^{m} x_{ij} = 1 \qquad j=1, 2, \ldots, n$

$0 \leq x_{ij} \leq y_i \leq 1 \qquad i=1, 2, \ldots, m$
$\qquad\qquad\qquad\qquad j=1, 2, \ldots, n$

$y_i = (0, 1) \qquad i=1, 2, \ldots, m$

where: $c_{ij} = t_{ij} * D_j$

t_{ij} = the unit transportation cost from facili
to customer j

D_j = demand at j

x_{ij} = the fraction of D_j supplied from facilit

F_i = the fixed charge associated with facilit

n = the number of customers

m = the number of proposed facilities

Equation (2-7) specifies that each customer must have a demands satisfied and inequality (2-8) requires that demand be satisfied by a facility unless it exists. Note that even small problem the above formulation has a formidable number straints and variables. For an N-customer, M-plant problem are N x M + N + M constraints and N x M + M variables. Thus the case M=10 and N=30, the problem to be solved has 340 con and 310 variables.

It would seem, at first glance, that if the integer req (2-9) were replaced by the simple constraint $y_i \leq 1$, the sol the resulting linear programming problem would be integer. a counter example is reported and the means of solution firs

empted is an integer programming cutting plane technique of ory (1958). Thia is ahown to have a very slow convergence even small problems, and is abandoned for larger problems. Balinski gests another scheme for solution which involves partitioning problem using a method of Benders (1962). This requires the cesaive solution of more complex integer programming problems. roof is supplied that the method will converge to the optimal ution in a finite number of ateps, but it appeara that computation e will be quite large for this type of aolution aa well.

Efroymson and Ray (1966) presented a method of solution to the e problem using a branch and bound scheme that has been tested putationally with good results. Their formulation of the blem is:

imize: $$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m} F_i\, y_i \qquad (2\text{-}10)$$

ject to: $$\sum_{i=1}^{m} x_{ij} = 1 \qquad j=1,\ 2,\ \ldots.,\ n \qquad (2\text{-}11)$$

$$\sum_{j=1}^{n} x_{ij} \leqslant n_i y_i \qquad i=1,\ 2,\ \ldots.,\ m \qquad (2\text{-}12)$$

$$x_{ij},\ y_i = (0,\ 1) \qquad (2\text{-}13)$$

re: c_{ij} = cost of supplying the entire demand area j from warehouse i

F_i = fixed charge for establishing warehouae i

y_i = 1 if the i'th warehouse is built, 0 if not

x_{ij} = 1 if the demand srea j is served b
i, 0 if not

n_i = maximum number of demand sreas tha
assigned to facility i

m = number of possible warehouse sites

n = number of demsnd areas

An examinstion of the constrsint set shows that Bsli
explicit constrsints on the vslue for each allocation to
(2-8) have been collapsed into s amaller set of constrain
maximum amount that may be sssigned to s facility (2-12).
straint specifies thst if the i'th fscility does not exis
no sssignments may be made to it. However, if it does ex
menta may be made to it up to the value of n_i. The value
becomes an important point in the development of the algo
solution method involves an implicit ssearch of all the po
feasible solutions. Suppose sll the feasible solutions w
sented as branches of a tree with each branch exhibiting
value, zero or one, for each variable. In order to svoid
the entire tree to find the optimal solution, means for e
the lower bound on the solution by continuing down the tr
available. If at any point on a branch the estimate of t
bound is greater than a known feasible solution, that bra
dominated and may be abandoned. The speed with which suc
procedure progresses to an optimal solution depends on th
used to obtain lower bounds and the rules used for determ
which branch to investigate next.

In establishing their branch and bound scheme to solve the problem, Efroymson and Ray had to specify a means for finding a lower bound at each node on a branch. They specified that this could be accomplished by solving the problem denoted by the obj tive function (2-10) plus constraints (2-11) and (2-12), while ignoring the integer requirement (2-13). If n_i is a capacity c straint such that it is limiting, i.e. areas that might assign are excluded because there is not enough capacity at i, the pro to be solved at each node requires the solution of a transporta problem. However, if n_i is greater than or equal to all the ar that could possibly assign to i, the lower bound problem at eac node may be solved by a very simple iterative procedure that i very fast. Simply assign the area to the open facility such th transportation cost plus expansion cost at the site is minimize In order to speed their algorithm, the authors abandoned the requirement for a capacity constraint on facilities by setting at a number greater than or equal to the total number of demand areas that could possibly assign to that facility. Solution of problems with 50 possible warehouse locations and 200 demand ar in 10 minutes on the IBM 7094 is reported.

Spielberg (1969), in a more recent paper, discussed largel the same problem. His algorithm for solution is also branch an bound, but contains many added features which shorten computati time. A great deal of computational experience is reported wit problems as large as 100 facilities and 150 demand areas. The

author also suggested extensions of the problem, but reported no computational experience for these.

Looking at the special problem of the location of transfer facilities for solid wastes, it is noted that there is an added dimension beyond that of the warehousing formulation. The facili to be located are intermediate points within a trans-shipment net work between the sources and sinks of flow. Thus the transportat cost involves two elements, transportation from source to intermediate point, and from intermediate point to the sink. There ar possible facility costs at the intermediate point as well as variable costs at the sources and sinks.

One of the weaknesses of all the previous work in warehousi problems is that there may be no capacity restrictions on the amo of flow through a central facility when in fact the cost estimat and the physical reality of the problem require that there be su a constraint. The problem of locating an intermediate facility capacity constraints on flow will henceforth be called the capac tated trans-shipment facility location problem. The only previo work on this problem is that of Skelly (1968). In addition to recognizing the above features of the problem, Skelly was also interested in time-staging of construction of facilities and dis points. He formulated the problem with a series of linear const which define the trans-shipment and time requirements. This lea a large fixed charge integer programming problem.

Skelly chose as his solution method an algorithm by Walker (

r the fixed charge problem. This algorithm is simplex-based, i.e requires that a basis the size of the constraint set be estab-shed and manipulated. Furthermore, it is a heuristic and may not eld a global optimum. The fact that the Walker algorithm is mplex-based leads to some problems in computational efficiency. ans-shipment problems belong to a class of network flow problems described by Ford and Fulkerson (1962), which may be written in near programming form but because of the sparseness and special ructure of the constraint matrix, can be solved much more quickly d efficiently by primal-dual labeling procedures. Because of the mplex approach, the size of the problem that Skelly may consider severely limited. Furthermore, since the value of such models in the repeated solution using different parameters to show nsitivity, the use of a heuriatic algorithm may be self-defeating

B. The Cspacitated Trans-Shipment Facility Location Problem

The remainder of this chapter is devoted to a new solution method for the capacitated trans-shipment facility location prob where there are fixed charges associated with the intermediate facilities. The algorithm guarantees an optimal solution and wi solve fairly large problems within moderate computation time. T problem to be aolved is stated as follows:

There is a set, K, of sources for a product with an amount at esch source. In addition, there is a set of sinks, J, for th product, each with an upper and lower bound on demand of D_j^u and A set of propoaed intermediate facility sites, I, has been sugge ss trans-shipment points between the sources and sinks. Each pr posed facility has a fixed charge, F_i, a variable unit cost, V_i which is s linear function of the amount shipped through the facility, and a capacity, Q_i. The problem is to find which fac should be built and which sources and sinks each facility serve that the total cost of facilities and trans-shipment is minimiz

In the context of a solid waste collection system, the flo through the system is solid waste material with the sources rep ting the locations of waste generation and the sinks representi the disposal methods and locations. A lower bound on the amoun solid waste reaching a sink is motivated by the fact that for s types of present disposal processes such as incineration, a min throughput is necessary to keep the process operating. Future

development in waste disposal technology may also lead to other processes which require continuous operation. The intermediate facilities are transfer stations where transfer may take place fr smaller collection vehicles to some other means of transport to t disposal points (sinks). The means of transport from the transfe station may be by vehicles such as large trucks or trains, or by non-vehicle methods such as pipelines. In addition to simple transfer, the activity which takes place at a transfer station ma include some form of processing such as sorting, compacting and/o incineration.

In mathematical form this problem becomes the following:

Problem One

$$\text{Minimize:} \quad \sum_{i=1}^{m} F_i y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} c^*_{ij} x^*_{ij} + \sum_{i=1}^{m} \sum_{k=1}^{p} c^{**}_{ki} x^{**}_{ki} \qquad (2-$$

subject to the constraints:

$$\sum_{i=1}^{m} x^{**}_{ki} \geq S_k \qquad k=1, 2, \ldots, p \qquad (2-$$

$$\sum_{j=1}^{n} x^*_{ij} = \sum_{k=1}^{p} x^{**}_{ki} \qquad i=1, 2, \ldots, m \qquad (2-$$

$$\sum_{k=1}^{p} x^{**}_{ki} \leq Q_i y_i \qquad i=1, 2, \ldots, m \qquad (2-$$

$$D_j^{\,u} \geq \sum_{i=1}^{m} x^*_{ij} \geq D_j^{\,l} \qquad j=1, 2, \ldots, n \qquad (2-$$

x^{*}_{ij}, x^{**}_{ki} are non-negative integers

$y_i = (0, 1)$

where: y_i = 1 if the i'th facility is built

= 0 otherwise

x^{*}_{ij} = flow of material from facility i to sink

x^{**}_{ki} = flow of material from source k to intermediate point i

$c^{*}_{ij} = c_{ij} + r_j$ = unit cost associated with a tr of material from facility i to (dollars per unit)

c_{ij} = unit shipping cost from facility i to sin (dollars per unit)

r_j = unit variable cost associated with using (dollars per unit)

$c^{**}_{ki} = c'_{ki} + t_k + V_i$ = unit cost associated with transfer of material fro source k to facility i (dollars per unit)

c'_{ki} = unit shipping cost from source k to facil (dollars per unit)

t_k = unit variable cost associated with using (dollars per unit)

V_i = unit variable cost associated with using facility i (dollars per unit)

F_i = fixed charge for establishing facility i (dollars)

S_k = amount supplied at source k

D_j^u = upper bound on amount demanded at sink j

D_j^l = lower bound on amount demanded at sink j

Q_i = capacity of the i'th facility

m = number of proposed fscility sites

n = number of demand aress

p = number of supply points

Inequslity (2-15) expresses the requirement that flow from the ce cannot exceed the supply of material there. Equstion (2-16) es the conservation of flow requirement that the flow entering i'th facility must be equal to the flow leaving it. In uslity (2-17) the fixed charge nature of facility location is essed. If the i'th facility does not exist, $y_i = 0$ and no flow take place through it. If the i'th facility does exist, $y_i = 1$ flow up to Q_i may psss through it. Inequality (2-18) specifies flow to sink j must be between the upper and lower bound on capacity of the sink.

There are several additionsl constraints or side conditions might be of interest in the sbove formulation. A budget con-int on the number or cost of the facilities to be built, or s al exclusivity constraint which allows a particular facility to only if another does not, might be included. As noted with s (disposal points) a lower bound requirement different from on the flow through s facility or a particular arc due to nical requirements might also be imposed. The side conditions be discussed in detail later in this chapter.

C. Method of Solution

It is desired to find a solution to Problem One that will gi values to the variables x_{ij}^{*}, x_{ki}^{**} and y_i so that the constraint se is satisfied and the objective function is minimized. One means doing this would be to set up the problem as an integer linear pr gramming problem and attempt to solve it by a cutting plane metho such as that of Gomory (1958). However, for an M-facility, N-demand-area, P-supply-point problem there are

$$2(M + N + P) \text{ constraints}$$

and $$M \times N + M \times P + M \text{ variables which are}$$

required to be integer.

Thus for a problem of M=10, N=10, P=10, the integer program would have 60 constraints and 210 variables. Solution of the corresponding linear programming problem with the integer constraints relaxed will not necessarily yield integer solutions.

However, there are some linear programming problems which ha special structure that may be related to a network, which may be solved optimally by procedures other than the simplex method of linear programming in a much shorter time and yield integer solu-tions. An example is the trans-shipment problem which represents supply-demand relationship where demands for a product at some points are satisfied by shipments from sources of the product through intermediate points. The resulting linear programming pr lem may be shown to have the form of a network, and finding the

imum cost maximum flow through the network gives the solution the trans-shipment problem.

Consider Problem One. It has some elements of network struc-e, but it may not be represented as a network problem. qualities (2-15), (2-16) and (2-18) exhibit an incidence matrix ch by themselves would form a trans-shipment problem and would ld integer solutions. The addition of inequality (2-17) which a coefficient of one variable that may be different from zero one, destroys this property. However, a related but similar blem may be written in a form that will allow it to be solved a network algorithm. Consider the problem of determining the w through the system from supply source through intermediate ilities to demand areas if the facilities to be built are known. s implies that the values for the y_i's (the variable which ermines which facilities are built) are known and given. Let S al the set of indices representing the values of $y_i=1$. Then problem may be written in the following form:

blem One (a)

imize: $$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{*} x_{ij}^{*} + \sum_{i=1}^{m} \sum_{k=1}^{p} c_{ki}^{**} x_{ki}^{**} + \sum_{i \in S} F_i \qquad (2\text{-}19)$$

ject to the constraints:

$$\sum_{i=1}^{m} x_{ki}^{**} \geq S_k \qquad k=1,\ 2,\ \ldots.,\ p \qquad (2\text{-}20)$$

$$\sum_{k=1}^{p} x_{ki}^{**} \leq Q_i \qquad i \in S \qquad (2\text{-}21a)$$

$$\sum_{k=1}^{p} x_{ki}^{**} = 0 \qquad i \notin S$$

$$\sum_{j=1}^{n} x_{ij}^{*} = \sum_{k=1}^{p} x_{ki}^{**} \qquad i=1, 2, \ldots., m$$

$$D_j^{u} \geq \sum_{i=1}^{m} x_{ij}^{*} \geq D_j^{1} \qquad j=1, 2, \ldots., n$$

$$x_{ij}^{*}, x_{ji}^{**} \geq 0$$

where the variables are defined in the same manner as for P

One, and

S = the set of indices for which y_i=1.

This problem has the form of a network problem as desc

Ford and Fulkerson (1962), and can be solved by finding the

cost maximum flow in the network. The method for finding t

cost maximum flow through the network is the out-of-kilter

which was developed by Fulkerson (1961).[1/] One of the comput

for the IBM 7000 and 360 series computers is in Fortran IV

and is available through the IBM Share Distribution System.[2

It should be noted that without the upper and lower bo

demand at the sink expressed in inequality (2-23), the abov

[1/] Those interested in following the development and proofs out-of-kilter algorithm are directed to Ford and Fulkerso or Fulkerson (1961). A good description of a more primar is found in Durbin and Kroenke (1967). A discussion of n techniques is found in Clasen (1968).

[2/] IBM Share Distribution #3536, "Out-of-Kilter Network or T tation Problem Solver", IBM, White Plains, New York.

would be a simpler trans-shipment problem rather than an out-of-kilter problem. However, the existence of the need for such constraints for this and other aspects of the problem means that an out-of-kilter problem rather than a simpler trans-shipment problem will be used for the basis for algorithmic development. The advantages of being able to formulate this subproblem as a network problem solvable by a network algorithm such as the out-of-kilter are twofold. First, network algorithms are very fast when compared to simplex based linear programming techniques. Second, the out-of-kilter algorithm may be started with any existing solution after constraints or costs have been changed. This restart capability will be of considerable interest in the solution scheme to be developed. Because of the network structure of the problem, both the network algorithm and a simplex algorithm will always yield integer solutions.

The network structure of a location problem has been recognized by other authors. Efroymson and Ray (1966), and Spielberg (1969) while working on simpler location problems noted this structure and took advantage of it in a branch and bound scheme. In their case the resulting structure was that of a transportation problem and the means of finding a lower bound on a branch might involve the solution of such a problem at each node of the branch. However, if the simplifying assumption is made that facilities do not have an upper bound on capacity, the transportation problem decomposes to a simple iterative scheme which is quite fast. Both authors

attempted to save time by ignoring capacity constraints and using the simple iterative scheme. This limited their work to transportation problems.

The technique to be proposed below arose from the need to include both a trans-shipment network structure, and capacity constraints on facilities in the formulation of transfer station location problems. It was clear from the outset that this more general formulation would require greater computation time, but it was anticipated that the increased cost of finding a solution would be more than offset by the greater generality of the solution. Much of tha success of the algorithm is based on the ability to restart the out-of-kiltar algorithm at each stage, so that little computational effort was necessary to calculate successive lower bounds. Trans-shipmant and transportation algorithms, with slight manipulation of the solution, also exhibit this restart capability.

A Sketch of the Algorithm

It is desired to solve Problem One, which is repeated here for nvenience.

blem One - The Capacitated Trans-shipment Facility Location Proble

$$\text{nimize:} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c^*_{ij} x^*_{ij} + \sum_{i=1}^{m} \sum_{k=1}^{p} c^{**}_{ki} x^{**}_{ki} + \sum_{i=1}^{m} F_i y_i \qquad \text{(2-24}$$

bject to the constraints:

$$\sum_{i=1}^{m} x^{**}_{ki} \geq S_k \qquad k=1, 2, \ldots, p \qquad \text{(2-2}$$

$$\sum_{j=1}^{n} x^*_{ij} = \sum_{k=1}^{p} x^{**}_{ki} \qquad i=1, 2, \ldots, m \qquad \text{(2-2}$$

$$\sum_{k=1}^{p} x^{**}_{ki} \leq Q_i y_i \qquad i=1, 2, \ldots, m \qquad \text{(2-2}$$

$$D_j^u \geq \sum_{i=1}^{m} x^*_{ij} \geq D_j^1 \qquad j=1, 2, \ldots, n \qquad \text{(2-2}$$

x^*_{ij}, x^{**}_{ki} are non-negative integers

$y_i = 0, 1$

ere the notation is as previously stated.

This problem has two elements that make it difficult to solve. rst, it is a fixed charge problem with the fixed charge variable, appearing in inequality (2-27) with a coefficient Q_i which may different from plus or minus one. Second, an integer solution

is required. However, as has been previously noted, if the
of the fixed charge variables, y_i, are known, then the resu
problem of finding the values of the flow variables, the x_i^*
the x_{ki}^{**}'s, becomes a form of trans-shipment problem whose s
may be found by using the out-of-kilter algorithm to find t
minimum cost maximum flow of the corresponding network. In
to solve Problem One, some approximations will be made.

The flow of materials through the i'th facility is giv

$$\sum_{k=1}^{p} x_{ki}^{**}$$

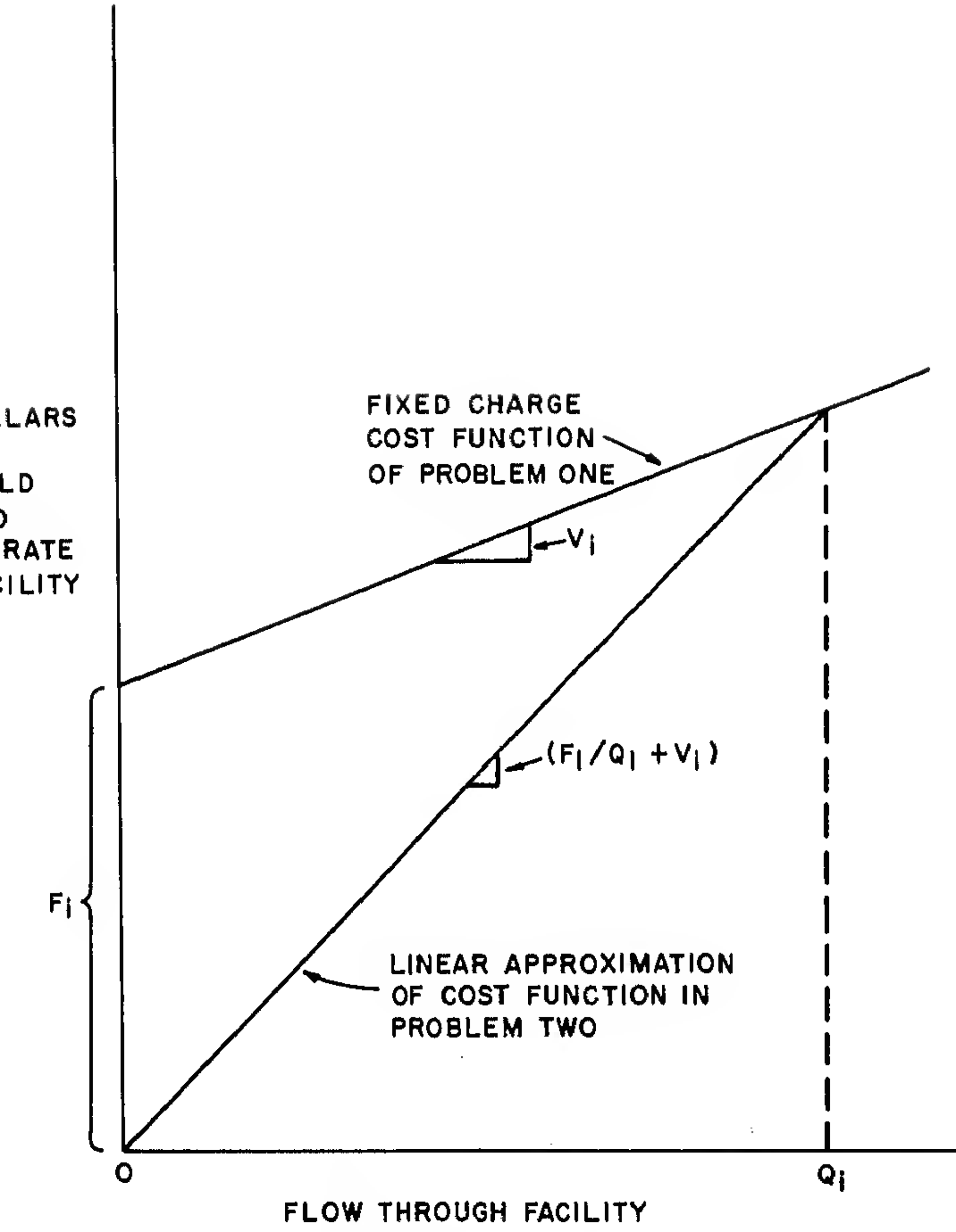
and the cost function of the i'th facility is

$$\begin{cases} F_i + V_i\left(\sum_{k=1}^{p} x_{ki}^{**}\right) & \text{if the i'th facility exi} \\ & \text{i.e. if } y_i=1 \\ 0 & \text{if the i'th facility doe} \\ & \text{exist, i.e. if } y_i=0 \end{cases}$$

The slope of this cost function is shown in Figure 2-1, alo
a linear approximation of the cost. The approximation is a
in the following manner. Convert the fixed charge, F_i, int
charge by dividing by the capacity of the facility Q_i. The
approximate cost function is

$$\left(\frac{F_i}{Q_i} + V_i\right)\left(\sum_{k=1}^{p} x_{ki}^{**}\right)$$

and it underestimates the true cost function given in (2-29
at two points where they are equivalent. These are when th



gure 2-1. The Fixed Charge Cost Function of Problem One and Its Approximation in Problem Two.

and the use of bounds for eliminating areas of search, th methods do differ somewhat in the search procedure. Impl enumeration implies that a branching scheme will be set u all possible feasible solutions will be enumerated either or explicitly. Bounds are used to avoid searching branch are obviously dominated by existing feasible solutions. such techniques are Glover (1965), Balas (1965), Balas (1 Geoffrion (1966, 1967). Branch and bound implies that th feasible solutions is divided into mutually exclusive set lower bounds estimated until an optimal solution has been and verified. In implicit enumeration, the goal is to en all solutions while in branch and bound the goal is to su into subsets until lower bound estimates show that an opt solution has been found. Examples of branch and bound te are the work of Land and Doig (1960) and Little (1963). of implicit enumeration in the field of environmental eng is a solution to a water pollution control problem by Lie Marks (1968).

The method of solution chosen to solve Problem One i and bound. The basic reasoning behind the technique is t possible to solve a highly structured problem by solving structured surrogates. A feasible solution to the surrog is always a feasible solution to the highly structured pr approximated as well. However, the value of the objectiv of the surrogate is not necessarily the value of the obje

nction of the problem it approximates with the same feasible lution. Because of the linear approximation used, the cost of e surrogate is lower than or equal to the cost of solving the iginal problem. Define an exact solution as an optimal, feasible lution to the surrogate where the value of the objective function r both the surrogate problem and the original problem are the me. A lower bound solution represents the case when the cost of e surrogate problem is lower than the cost of the original oblem. The surrogate problems are generated in such a manner at the solution space for the original problem is subdivided into tually exclusive collectively exhauative sets. The solution of e problem representing each set is a lower bound for that set. e optimal solution is found when the smallest of the lower bounds r all the sets is an exact solution to the original problem.

Consider solving Problem One. The less constrained Problem Two ll be solved as a surrogate for it. A solution is generated by e out-of-kilter algorithm and the cost and exactness of the solu- on to Problem One are investigated. If, for each facility the ow through it is exactly zero or the capacity of the facility, an act solution to Problem One has been found as well. Since this lution represents the only lower bound estimate of Problem One, d it is feasible to Problem One, the optimal solution of the iginal problem has been found as well. The algorithm terminates. wever, suppoae that for some facility, i^*, the flow is greater an zero but less than capacity. Choose this facility to branch on

In all feasible solutions to Problem One only two situat
occur regarding i^*. Either $y_i{}^*=1$ and the i^* facility ex
$y_i{}^*=0$ and the i^* facility does not exist. Construct two
problems: Problem Two(a), (i^*), is Problem Two plus the
$y_i{}^*=1$. In order to set up this problem, the linear appr
associated with i^*, $\frac{F_i{}^*}{Q_i{}^*} + V_i{}^*$, is replaced by its true
is a constant, $F_i{}^*$, plus a variable cost of $V_i{}^*$. No cha
in the constraint matrix since it has already been assum
$y_i=1$ in the original formulation of Problem Two. Since
constant, it is simply added to the cost of the solutior
Problem Two(a), (i^*), found by solving the resulting net
the out-of-kilter algorithm. Since Problem Two(a), (i^*)
constrained than Problem Two, its cost must be greater t
equal to the cost of Problem Two. Note that the flow th
facility may take on any value between zero and capacity
correct cost will be assigned, since the fixed charge fo
facility is included in the cost, and the variable charg
function of flow through the facility. Now construct Pr
i^*, which is Problem Two plus the constraint $y_i{}^*=0$. Thi
handled in two ways. Either set the variable cost at fa
infinity or set the capacity at facility i^* to zero. Bo
dures will produce the result of blocking flow through t
The solution to the two problems represents lower bounds
mutually exclusive sets of feasible solutions to the ori
The lower bound with the lowest objective function is ex

exactness to Problem One. If it is exact, it is optimal. If

a new variable, i^{**}, is chosen for branching, and two new prob

are set up. One problem has $y_{i^*}=1$, $y_{i^{**}}=1$, and the other has

and $y_{i^{**}}=0$. The cost of these solutions, along with the remai

lower bound from the first branch, are searched for a minimum

more, and the exactness check is made. The procedure is conti

until the optimal solution is found. The procedure must termi

since the process of adding constraints continues only until t

enumeration of all possible feasible solutions has been found.

To present the subproblem solution in a general form, def

S^* = the set of all indices representing facilities

S^1 = the set of facilities constrained to be built (i.e. $y_i=1$)

S^o = the set of facilities constrained not to be bui (i.e. $y_i=0$)

$\underline{S}$ = the set of facilities not yet constrained

Then $S^* = S^1 \cup S^o \cup \underline{S}$

and $S^1 \cap S^o \cap \underline{S} = \phi$;

that is, the sets are mutually exclusive and totally exhaustiv

Then a general subproblem may be written as:

Minimize:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c^*_{ij} x^*_{ij} + \sum_{k=1}^{p} \sum_{i=1}^{m} c^{**}_{ki} x^{**}_{ki} + \sum_{i \in \underline{S}} \frac{F_i}{Q_i} \left(\sum_{k=1}^{p} x^{**}_{ki} \right) +$$

$$\sum_{i \in S^o} \infty \times \left(\sum_{j=1}^{n} x^*_{ij} \right) + \sum_{i \in S^1} F_i$$

subject to the constraints:

$$\sum_{i=1}^{m} x_{ki}^{**} \geqslant S_k \qquad k=1, 2, \ldots, p$$

$$\sum_{k=1}^{p} x_{ki}^{**} \leqslant Q_i \qquad i \in \underline{S} \cup S^1$$

$$\sum_{j=1}^{n} x_{ij}^{*} = \sum_{k=1}^{p} x_{ki}^{**} \qquad i \in S^*$$

$$D_j^u \geqslant \sum_{i=1}^{m} x_{ij}^{*} \geqslant D_j^1 \qquad j=1, 2, \ldots, n$$

$$x_{ij}^{*},\ x_{ki}^{**} \quad \text{are non-negative integers}$$

snd all symbols are as previously defined.

The ease of keeping track of the various solution
puter is aided by the use of associative lists which a
to be kept in order of increasing cost, and makes loca
lowest cost solution relatively simple.

The choice of a variable on which to branch can m
deal of difference in the speed of finding the solutio
used in the procedure for solving Problem One involves
the facilities which have flow greater than zero but l
capscity of the facility. Denote the fraction of flow
facility i as f_i (defined as the flow divided by the c
Select for branching that facility whose value of f_i i
to 0.5, which is the rule that Land and Doig (1960) su

5. A Detailed Algorithm for Solving the Capacitated Trans-Shipment Facilities Location Model

1. The following data are given:

   m = the number of proposed facility sites

   n = the number of demand areas

   p = the number of sources

For each facility site, i, the following are known:

   F_i = the fixed cost of establishing the i'th facility

   V_i = the unit variable cost of operating the established facility

   Q_i = the capacity of the facility

For each demand area, j, the following are known:

   D_j^u = the upper bound on quantity demanded at the j'th demand area

   D_j^l = the lower bound on quantity demanded at the j'th demand area

   r_j = unit variable cost of satisfying demand at j

For each supply point, k, the following are known:

   S_k^u = the upper bound on quantity supplied at point k

   S_k^l = the lower bound on quantity supplied at point k

   t_j = unit variable cost of providing supply from point k

In addition, the m x n matrix c_{ij}, and the p x m matrix c^*_{ki} are assumed known where

   c_{ij} = the unit cost of transportation from facility i to demand area j

   c^*_{ki} = the unit cost of transportation from source k to facility i

2. Initialization:

Define S = the set of all indices of facilities
have been fixed at zero or one

$\underline{S}$ = the set of all indices not yet set

S^* = the set of all indices = (1, 2, 3, ..

Then $S \cup \underline{S} = S^*$ $S \cap \underline{S} = \phi$ (the empt

Initialize $S = \phi$

$\underline{S} = S^* = (1, 2, 3, \ldots, m)$ = all indi
senting

3. Construct a graph in the following manner: There wi
sets of nodes (I, J, K) which make up the graph, plu
node S#. The graph will be in circulation form[3/] so
source = sink = S#.

Define I = (set of nodes which represent the faci
alternatives)

For each of the m sites, a capacitated node is
This is accomplished by representing the site
with a directed arc joining them (see Figure 2
upper bound capacity of the arc is the capacit
the facility it represents. The lower bound c
zero. The unit cost on the arc is Z_i, which w
defined later.

Define J = (set of nodes which represent the dema

Define K = (set of nodes which represent the supp

3/ Circulation form means an arc exists from the sink
the source.

Arcs connecting the nodes:

<u>From S# to set K</u>: Define a directed arc from S# to each node in K. The unit cost on the arc is t_k. The upper bound on capacity of the arc is $S_k{}^u$ and the lower bound is $S_k{}^l$. Assume the upper bound is equal to the lower bound.

<u>From set K to set I</u>: Define a directed arc from each node in K to the receiving end of each capacitated node in I. The cost of the arc is c^*_{ki} and the upper and lower bound capacity of the arc may be set as desired. Assume the upper bound is infinity and the lower bound is zero.

<u>From set I to set J</u>: Define a directed arc from the transmitting end of each capacitated node in I to each node in J. The unit cost of the arc is c_{ij}, and the upper and lower bound capacity may be set as desired. Assume the upper bound is infinity and the lower bound is zero.

<u>From set J to S#</u>: Define a directed arc from each node in J to S#. The unit cost on the arc is r_j, and the upper bound and lower bound capacities are respectively $D_j{}^u$ and $D_j{}^l$.

Figure 2-3 shows a graph representation for m=3, n=3, k=2.

. Establish the unit cost, Z_i for the capacitated nodes in set

Define y_i = a decision variable

Define y_i = 1 if the i'th variable is in the set S and if the i'th facility is to be built

Define y_i = 0 if the i'th variable is in the set S
the i'th facility is not to be built
if the i'th variable is in the set $\underline{S}$

Define $Z_i = 0$ if $y_i=1$ and $i \epsilon S$

Define $Z_i = \infty$ if $y_i=0$ and $i \epsilon S$

Define $Z_i = \frac{F_i}{Q_i}$ if $y_i=0$ and $i \epsilon \underline{S}$

5. Find the minimum cost maximum flow through the graph
out-of-kilter algorithm. Define this cost as CGRAPH
is the first time through this step and the attempt
the graph using the out-of-kilter method indicates n
solution, no feasible solution to the problem exists
If not, continue.

6. Define the total cost of the solution, COST, as

$$COST = CGRAPH + \sum_{i \epsilon S} F_i y_i$$

7. Examine the solution to see if it is exact:

Define X_i as the flow through the capacitated nod
representing the i'th facility

The solution is exact if for all $i \epsilon \underline{S}$

$$X_i = 0 \quad \text{or} \quad X_i = Q_i$$

If the solution is not exact, find the index of c
members of the set $\underline{S}$ for which $0 < X_i < Q_i$, as th
candidate for branching. Select that member for
nearest to 0.5.

re information about the solution. File on a list ranked on

cost of the solution information about the exactness, the

bers of the S set and the values to which they are set, and

next candidate for branching.

ove the first problem on the list, i.e. the lowest cost

ution found so far. If the solution is exact, it is

imal. TERMINATE.

the solution is inexact, set up subproblems (a) and (b):

(10a). For subproblem (a):

Set $y_i=1$ for the index stored as the candidate for branching and switch that index from $\underline{S}$ to S

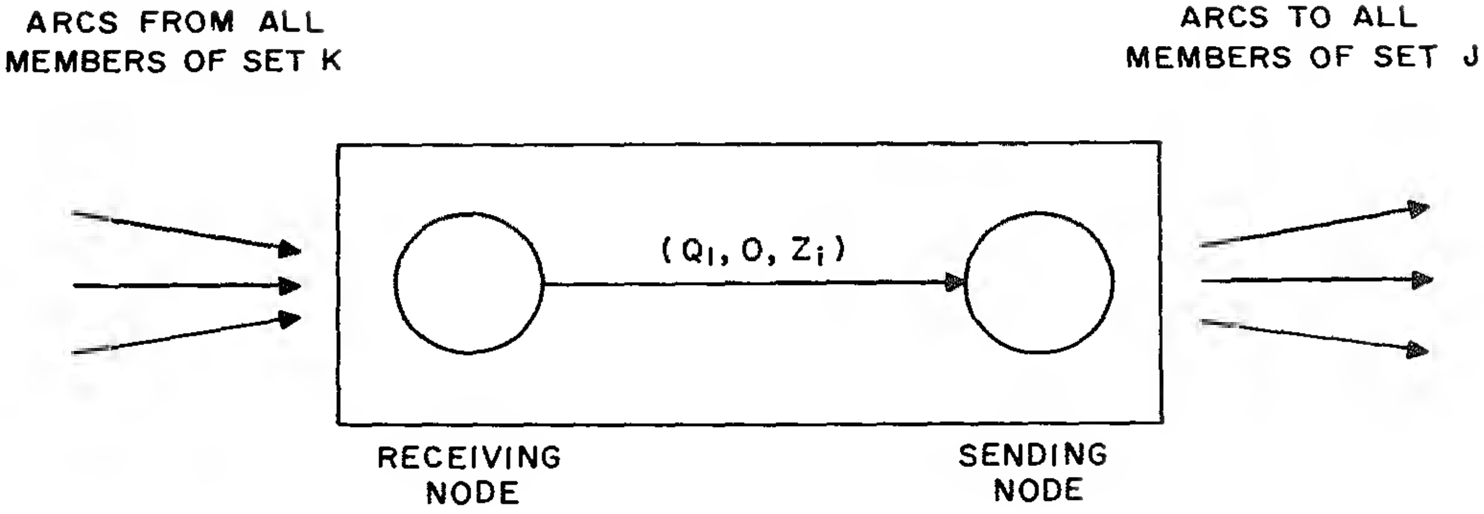
Then do steps 4 through 8 and return to do subproblem (b) in (10b)

(10b). For subproblem (b):

Set $y_i=0$ for the aame index as in (a) and switch that index from $\underline{S}$ to S

Then do steps 4 through 8.

Go to step 9.



Q_i = UPPER BOUND ON FLOW THROUGH FACILITY i.

Z_i = UNIT COST ASSOCIATED WITH USING FACILITY i.

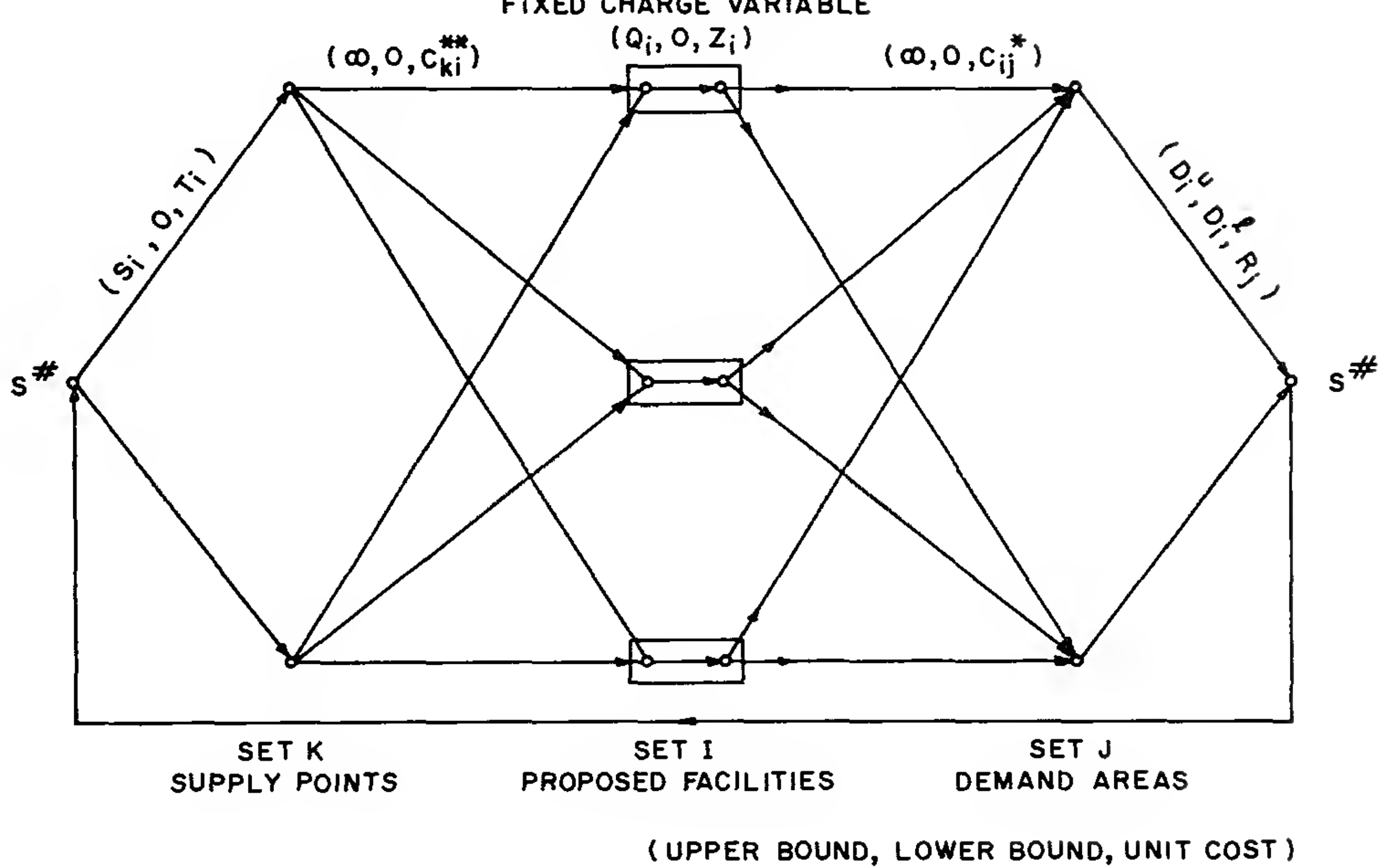


Figure 2-3. A Graph Representation for a Problem with m=2; n=3, k=2.

F. Computational Results Using the Algorithm

The algorithm as described was coded in ASA Fortr
executed on an IBM 7094 computer. The out-of-kilter alg
to solve the lower bound problem at each branch was a Sh
Distribution Program written by Clasen; the list process
routine package used in keeping track of the various sol
written by Bellmore (1966). In Chapter V results based
of data from example problems in solid waste collection
General impressions of this type of algorithm gained by
with randomly generated data are as follows: The usual
branch and bound algorithms is that they are data depend
same size problem with different data can lead to very d
running times. In the case of the algorithm tested, the
that ran the fastest for a given size were those where t
costs were sufficiently high so that only a few faciliti
built. From computational experience with such algorith
been found that problems where the optimal solutions are
either a few or all of the facilities, generally run the
Problems where there is a delicate trade-off between fac
transportation costs, so that several facilities are bui
take longer to derive optimal solutions. Further, the r
out-of-kilter problems solved in obtaining the optimal a
definitely exhibited the value of the algorithm's restar
As the number of out-of-kilter solutions increased, the
solution decreased.

A possible limiting factor on the use of this algorithm is the amount of storage space needed within the computer for a lem is quite large.

Let F = the number of proposed facilities

D = the number of demand areas

S = the number of supply points

the number of arcs and nodes in the resulting graph is:

$$\text{Number of arcs} = D(1 + F(1 + S))$$

$$\text{Number of nodes} = D + 2F + S$$

naximum size problem that could be run on the 7094 with its 58 word memory was 300 nodes and 1500 arcs. The number of ible alternatives is dependent on the number of facilities g investigated. However, comparing two solutions with the number of facilities but different numbers of supply points show that there is a significant difference in time to solve out-of-kilter algorithm due to its larger number of arcs. The utational results are shown in Table 2-1.

Table 2-1.

Results of Trial Runs of the Algorithm with Randomly Generated

F	D	S	No. of Alternatives	No. of Arcs	No. of Nodes	No. of OKA's**	70 Ti (min
6	12	2	64	104	27	8	0.
12	10	2	4,096	194	35	8	0.
* 10	30	3	1,024	382	73	8	0.
* 10	30	2	1,024	382	53	8	0.
15	20	3	32,000	388	54	3	0.
30	20	3	10^{10}	733	84	15	1.
10	20	3	1,024	293	44	8	0.
20	20	3	10^{6}	503	64	6	0.
30	25	2	10^{10}	866	88	30	1.
+ 35	10	3	10^{12}	478	84	30	1.
+ 35	10	3	10^{12}	478	84	11	0.
+ 35	10	3	10^{12}	478	84	6	0.
50	10	3	4×10^{15}	713	114	20	1.

* These two solutions have decreasing fixed costs (in order gi

\+ These three solutions have increasing fixed costs (in order

** Number of times out-of-kilter algorithm applied.

G. Augmenting the Algorithm to Consider Additional Side Condit

There are several additional constraints that might also added to the problem to give more generality. Consider the fol conditions and the way the algorithm must be adjusted to allow their inclusion.

1. Mutual exclusivity constraints of the form of

$$\sum_{i \in E} y_i \leq 1 \qquad (2$$

where E is some subset of facilities for which only one member subset may be constructed. This constraint is used in the case the set E represents several treatment alternatives that might built at a possible site. Then either none or one of the possi alternatives may be built. It might also be used as an informa budget or political constraint. If one facility is built at on site, another cannot be built at a different site. The way tha this is handled in the algorithm is as an additional feasibilit requirement. In order to be an exact feasible solution to Prob One, the solution to Problem Two must not only satisfy flow thr facility constraints but the mutual exclusivity constraints as If it is violated, an exact solution has not been found. A fas convenient way to set up a branching rule for this procedure is following: Suppose that the variable chosen for branching is i and $i^* \in E$. Since no other member of E may appear in a feasibl tion if $y_i{}^*$ is set to one, set $y_i{}^* = 1$ and $y_i = 0$ for all $i \in E$ not equal to i^*. Switch all indices $i \in E$ from $\underline{S}$ to S.

2. Budget constraints of the form

$$\sum_{i \in W} a_i y_i \leq B \qquad (2\text{-}43)$$

where W may be a subset of the facilities or the entire set. The mutual exclusivity constraint in (2-42) is a special case of (2-43) (with all $a_i = 1$, and $B = 1$). Each time a member of the set E is a candidate for branching, a check is made to see if establishing that facility will violate the budget constraint. If it will not, then branching may proceed. If it will, the variable is only allowed in the solution at a value $y_i = 0$, and the search for another variable on which to branch is carried out.

3. Upper and lower bounds on flow through a facility if it exists. Equipment may exist for which a certain minimum use is necessary to keep it from deteriorating. The constraint takes the form

$$Q_i^{\,l}\, y_i \leq \sum_{k=1}^{p} x_{ki}^{**} \leq Q_i^{\,u}\, y_i \qquad (2\text{-}44)$$

where $Q_i^{\,l}$ and $Q_i^{\,u}$ are respectively the lower and upper bound on the capacity of the facility if it exists. The change in the algorithm that will allow this condition is in step 4. When Z_i, the cost of the arc, is defined, the capacity of the arc is defined as well.

Substitute these rules in step 4.

$Z_i = V_i$ and the upper and lower bounds are Q_i^u and Q
if $i \in S$ and $y_i = 1$

$Z_i = \infty$ and the upper and lower bounds are zero
if $i \in S$ and $y_i = 0$

$Z_i = \frac{F_i}{Q_i} + V_i$ and the upper and lower bounds are Q_i^u and z
if $i \in \underline{S}$

4. Upper and lower bounds on individual arcs such as

$$G_{ij}^l \leq x_{ij}^* \leq G_{ij}^u \qquad (2$$

may be added without any change in the algorithm since the out-kilter algorithm allows such restrictions on arcs.

CHAPTER III. LARGE SCALE SINGLE AND MULTICOMMODITY ROUTING IN GIVEN NETWORKS

Introduction

In this chapter, the problem of routing a flow of commoditie from sources through intermediate points to sinks in given networks is approached. As with the facility location problem of Chapter II the assumption is that the capacity of the vehicle which comprises the means of transport is smaller than the quantity of material to be moved, so that the small scale collection problems of routing between sources for collection, or between sinks for distribution, may be avoided. In the narrowest sense, finding the flow through a given network is a simplification of the facility location problem and may be readily solved by the use of a transportation or transshipment algorithm. However, there are often additional considerations that might be introduced to the problem which increase the generality of the formulation and the difficulty of finding solutions. The first such consideration is the addition of time and capacity constraints on vehicles, budget constraints on certain types of expenditures and constraints on manpower allocation. Work in the general area has been done by Garvin et al. (1957) and Barton (1967), and work in the specific area of solid waste collection has

al commodities. The work of Szwarc (1967) on multi-ansportation problems has been extended in thia and will be discussed. Another area of interest is nsidered by Ford (1958), Bomberault and White (1968) 968) called the dynamic trans-shipment problem, in desired to route vehicles not only in a spatial sense poral sense as well. This work finds application in ng of empty railroad cars and may have application in collection.

A. Literature Review

The earliest report in the literature of a vehicl problem through a given network with added conditions v Garvin et al. (1957) in an article showing applications programming in the oil industry. Included in the proble was that of routing vehicles from bulk refineries to se stations. The problem statement is as follows:

There is a given set of stations, k, each with a product D_k, and located at known distances from the bulk and from all other stations. There are several differen trucks available for delivering to the stations, each wi C_s, a given unit cost of operation and a given number o time period when that type of truck may be used. The p find the delivery schedule that will meet the demands fo at the stations within the capacity and time constraint minimizing the transportation costs.

For thia particular problem, it is assumed that demands are greater than any of the vehicle capacities. presented a linear programming formulation as follows:

Minimize: $$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{s=1}^{p} c_{ijs} x_{ijs}$$

subject to: $$\sum_{i=1}^{n} y_{ijk} = \sum_{u=1}^{n} y_{juk}$$ k=1, 2, ..., n
j=1, 2, ..., n
but $k \neq j$

$$\sum_{i=1}^{n} y_{ikk} = D_k \qquad k=1, 2, \ldots, n \qquad (3\text{-}3)$$

$$\sum_{k=1}^{n} \sum_{j=1}^{n} y_{ojk} = \sum_{k=1}^{n} D_k \qquad (3\text{-}4)$$

$$\sum_{i=1}^{n} x_{ijs} = \sum_{u=1}^{n} x_{jus} \qquad \begin{array}{l} j=1, 2, \ldots, n \\ s=1, 2, \ldots, p \end{array} \qquad (3\text{-}5)$$

$$\sum_{k=1}^{n} y_{ijk} \leq \sum_{s=1}^{p} C_s x_{ijs} \qquad \begin{array}{l} i=1, 2, \ldots, n \\ j=1, 2, \ldots, n \\ \text{but } k \neq j \end{array} \qquad (3\text{-}6)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} h_{ijs} x_{ijs} \leq h_s \qquad s=1, 2, \ldots, p \qquad (3\text{-}7)$$

$$x_{ijs},\ y_{ijk} \quad \text{are non-negative integers} \qquad (3\text{-}7a)$$

c_{ijs} = cost per unit for shipping from i to j in vehicle type s

x_{ijs} = the number of vehicles of type s sent from i to j

y_{ijk} = the quantity shipped from i to j with final destination

D_k = quantity demanded at k

C_s = carrying capacity of s'th type vehicle

h_{ijs} = time required to go from i to j in vehicle type s

h_s = time that an s type vehicle can be used

n = number of customers (or cities)

p = number of vehicle types

Expression (3-1) specifies the minimization of the cost of

vehicle operation. Equations (3-2), (3-3) and (3-4) ar
trans-shipment constraints and exhibit an incidence mat
Equation (3-2) specifies that all material shipped into
destined for sink k must equal the amount of material s
of the node destined for sink k. Equation (3-3) is a d
constraint which sets the demand at the k'th station, a
(3-4) is a source constraint specifying the amount to b
Equation (3-5) expresses a node constraint on trucks by
that the number of trucks of type s that enter a statio
the number that leave it. Inequality (3-6) relates mat
shipment to the capacity available to ship it, while in
specifies a time constraint on the availability of diff
of vehicles.

The authors have ignored the integer constraints
solved the problem by using a linear programming code w
of variables that take non-integer values. The problem
becomes unwieldy for any reasonable number of sources s
For n sinks and s different types of service vehicles t

$$2n^2 + ns \text{ constraints}$$

and

$$n^3 + n^2 s \text{ variables.}$$

A 10 sink problem has more than 200 constraints and 110
The authors did note the network structure of part of t
set and suggested that some improvement in running time
gained by exploiting it. So far, however, nothing furt

ared in the literature. The linear programming approsch con-es to appear as bigger, fsster codes become available. Barton 7), in work on aircraft scheduling and routing, wrote essentially same formulation and used the same technique to solve it. In case, C.E.I.R.'s LP/90 code and an IBM 7094 allowed him to e a 202 row, 515 column problem in 16 minutes. The solution is rted to have non-integer elements which are rounded to integer.

In application of models of this form to problems of solid e collection, the sources are the locations from which solid e is to be collected, the sinks are disposal points and the rmediate points are transfer stations where some form of pro-ing such as incineration or compacting might take place in tion to the transfer operation. Arcs joining these points esent alternative routes or methods by which the materisl may from sources to sinks. Solving the problem will show the over-cost of collection and which alternative routes are used to in it. Some authors have looked at the problem of modeling the all collection system in this manner. Wolfe and Zinn (1967) rted to linear programming to solve the problem for a small ple problem. The model they proposed would be an ordinary trans-ment model which might then be solved by a primal-dual algorithm kly, except for problems that arise in the representation of nerators. Any waste flow that enters this node is decomposed several different forms whose amounts are given proportions of inflow into the node. This means that flows leaving the node

have coefficients between zero and one and an incidence
longer exists. Since there are advantages to using a p
algorithm over a aimplex algorithm, Anderson (1968) pro
of-kilter algorithm for solving this problem which allo
nodes to be separated into given proportions. He showe
examples solved by hand and did not indicate if the alg
been coded. Thus, while it would be expected that such
would be much faster than solving the same problem by a
linear programming code, there is no evidence yet to su

In many cases in vehicle routing, it is desired
the use of a common vehicle fleet to move more than one
between supply and demand points for a commodity. This
larly true in solid waste collection, since many differ
of wastes such as residential, industrial, construction
materials, ashes, leaves and even Christmas trees are p
collected separately. Adding multicommodity aspects to
formulations as Garvin et al. (1957) proposed for singl
flow may be done quite simply. Additional constraints
ahipment network requirements for the additional commod
the commodities interact in their competition for shipm
are easily formulated and added to the problem. Howeve
formulation is already so complex that even a small pro
unmanageable in terms of computation technique and time
of more commodities makes the formulation unworkable.

To be able to deal efficiently with a multicommo

em it will be necessary to drop all consideration of additional raints such as those on time, vehicles and work rules which e the simple network structure of the problem. This leads to ormulation of multicommodity transportation or trans-shipment ems and to solution methods which will prove faster than linear mming using the simplex method. Szwarc (1967) formulated and ted a solution technique for a multicommodity transportation em which he calls the truck assignment problem. He formulated the following manner:

There exist m production sites each capable of producing p ities, and n demand locations where there is a demand for each commodities. The demand for a commodity is such that the capacity of a vehicle is allocated to only one commodity and tined for one demand location only. Thus the problem of g a vehicle between supply locations or demand locations is d and a vehicle's assignment will consist of loading a commo- t a production source, delivering its contents to one demand on and returning empty to a production source for reassignment. jective is to find the vehicle assignments that will minimize tal distance traveled while meeting the supply-demand constraints.

This problem, because of the requirements that a vehicle ing from source to sink may carry only one commodity and that turn from the sink to the source is done empty, is a far easier, omplicated problem than the general multicommodity flow problems ered by Ford and Fulkerson (1958). Each arc in the traditional ommodity flow problem may carry many different commodities at

the same time. But with the special formulation, eac source of a commodity to a sink of s commodity may ca commodity. Thus the problem may be decomposed into s simpler single commodity problems in which losded veh shipped from sources to sinka, and empty vehicles fro to sources. Arcs representing flow between sources a different commodities are not allowed. However, arcs back to the sources repreaent empty trucks and any sc reached from any sink.

Szwarc presents no mathematical formulation of but proceeds to a solution method based on solving tw transportation problems for an enlarged network in wh of several commodities becomea a diatinct source for commodities. Sinks are treated in the aame manner. production sources each supplying p commodities and r then he solves first an mp by np transportation probl routing full trucks from sources to demand aites, wit tween supply and demand for different commodities blo Then, the problem of routing the empty vehicles back is preaented as another tranaportation problem. A pr ahown which reconciles the two solutions into s set o assignments. A proof of optimality is included in th is a very quick, efficient method since solution time transportation problems ia quite small. A later sec chapter givea an extension to multicommodity trans-sh

the type described by Szwarc with the addition of many
nsiderations which make the formulation more general.

Dynamic trans-shipment networks deal with the problem of th vement of goods and vehicles from location to location over time aling with networks where there are time as well as spatial pects to the problem often changes the type of analysis performe e concern may shift from routing to finding the maximum flow rough a point that may occur in a given time sequence. The uting of vehicles must specify not only which arcs of the networ e traveled, but in what time period as well.

The first work to appear in the literature on dynamic flow oblems was by Ford (1958) and is also reported in the textbook b rd and Fulkerson (1962). The problem was to find the maximum namic flow through a given network where the dynamic flow at a int is defined as the total flow that passes that point during a ecified time period. The normal concept of a capacity of an arc a static or time-independent problem becomes a capacity per uni me in the dynamic problem and the cost on the arc represents the avel time through it. Ford showed that there is a direct relati ip between static and dynamic problems and that the maximum flow rough the natwork as obtained by a trans-shipment algorithm coul used directly to construct the solution to the dynamic problem. mberault and White (1968) were interested in the problem of rout pty shipping containers from sources where they exist to demand r them. In this case damand exists in only specified time perio

and there is both a travel time and a cost associated with t along an arc. The authors wished to apply a travel cost to between nodes as well as a time constraint and then minimize cost of shipping the containers to meet the temporal demand them. By enumerating the feasible routes that exist for a c tainer to reach a demand point before or as it is needed, th construct an expanded network through which flows which mini transportation costs may be found by the use of a trans-ship algorithm, much in the spirit of the Ford and Fulkerson algo White (1969) suggested an improved algorithm for solving the container problem, which involves decomposing into a nested of subproblems and usea an out-of-kilter algorithm for solut

Applications of the work on dynamic trans-shipment ne to problems of solid waste collection are of interest partic in the scheduling of large scale vehicles used for transport transfer sites to disposal areas. Such vehicles may be eith or railroad cars, and questions such aa how many of what typ needed may be approached using this formulation.

ormulation snd Solution of a General Multicommodity Truck

ssignment Problem

Szwarc (1967) described a method for solving multicommodity portation problems which is discussed in the preceding section. ork is limited in application because it csnnot deal with trans- ent networks. In the paper on his method, Szwarc did not give hematical formulation for the problem he was sttempting to . However, such a formulation gives a hint as to how it may tended to a far more general form, and solved optimally using t-of-kilter algorithm.

Consider the following definitions:

x_{ijk} = the number of vehicle loads of commodity k to be sent from production source i to demand site j

x^*_{ji} = the number of empty vehicles sent from demand site j back to production source i

c_{ijk} = cost per vehicle load of shipping commodity k from i to j

c^*_{ji} = cost per vehicle of returning an empty vehicle from demand site j to production source i

D_{jk} = demand in vehicle loads for commodity k st demand site j

S_{ik} = supply in vehicle loads for commodity k st production source i

m = number of production sources

n = number of demand sites

p = number of commodities

Then the general mathematical formulation for Szw

delivery problem is:

Minimize: $$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} c_{ijk}x_{ijk} + \sum_{j=1}^{m} \sum_{i=1}^{n} c^{*}_{ji}x^{*}_{ji}$$

subject to: $$\sum_{j=1}^{n} x_{ijk} \leq S_{ik} \qquad \begin{matrix} i=1,\ 2,\ \ldots., \\ k=1,\ 2,\ \ldots., \end{matrix}$$

$$\sum_{i=1}^{m} x_{ijk} \geq D_{jk} \qquad \begin{matrix} j=1,\ 2,\ \ldots., \\ k=1,\ 2,\ \ldots., \end{matrix}$$

$$\sum_{k=1}^{p} \sum_{i=1}^{m} x_{ijk} = \sum_{i=1}^{m} x^{*}_{ji} \qquad i=1,\ 2,\ \ldots.,$$

$$\sum_{k=1}^{p} \sum_{j=1}^{n} x_{ijk} = \sum_{j=1}^{n} x^{*}_{ji} \qquad i=1,\ 2,\ \ldots.,$$

x_{ijk}, x^{*}_{ji} are non-negative integer

The objective function in (3-8) is to minimize th

loaded trips from source to site and of empty return tri

Inequality (3-9) requires that the amount of commodity k

from source i to all sites must not exceed the supply of

at that source. Similarly, inequality (3-10) specifies

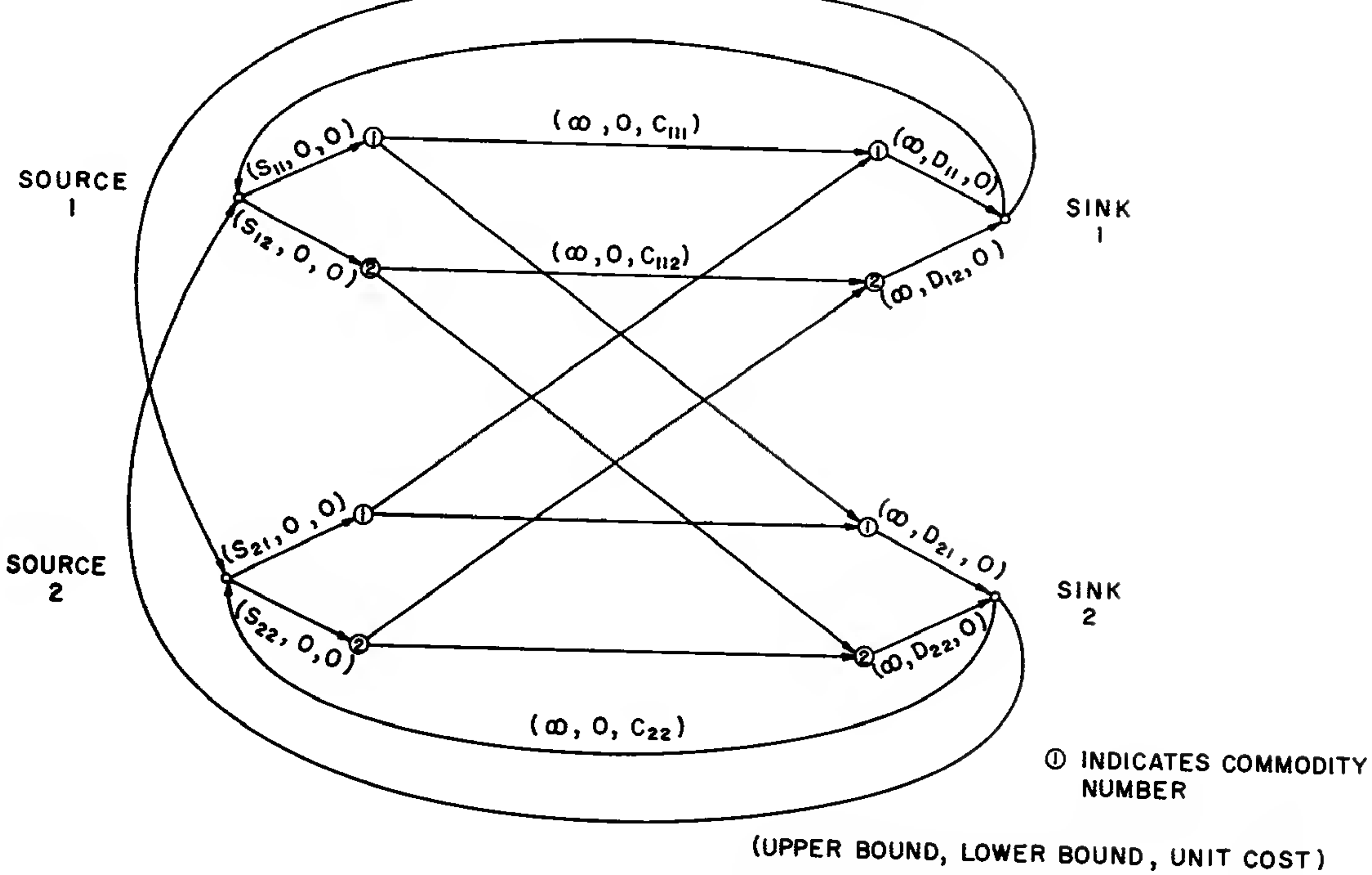
total amount of commodity k shipped to j from all source

greater than or equal to the demand for commodity k at t

Equations (3-11) and (3-12) are flow equations requiring

number of empty vehicles sent out of a demand site must equal
er of loaded vehicles arriving there, and that the number of
ehicles sent out of a source must equal the number of empty
arriving there. Inherent in this formulation is that
must return to a source after leaving a demand site. The
requirement (3-12a) will be shown to be automatically satis-
ce the problem may be set up in network form and the
found by finding the minimum cost-maximum flow using the
ilter algorithm. Figure 3-1 shows the graph corresponding
s problem for the case of m=2 sources, n=2 sinks, p=2 commo-

e computational difference between the two methods is shown
act that the Szwarc method requires the solution of two 4x4
tation problems and some manipulation with the solution.
lution by the out-of-kilter algorithm a graph of np + mp +
arcs and (m+n)(p+1) nodes (or, for the example case,
and 12 nodes), must be solved, which will take longer to
There are several advantages of the out-of-kilter approach
ke it a recommended procedure for general cases, however.
It is possible to consider any sort of routing network
ermediate facilities. In this case, it may easily be shown
ing intermediate node equations to the model formulation
aves a graph solvable by the out-of-kilter algorithm, but
zwarc's algorithm. Since most vehicle routing systems, in-
solid waste systems, involve the consideration of intermediate
uch an extension is of considerable analytic value.


SOURCE 1
SOURCE 2
SINK 1
SINK 2
(S11, O, O)
(S12, O, O)
(S21, O, O)
(S22, O, O)
(∞, O, C111)
(∞, O, C112)
(∞, O, C22)
(∞, D11, O)
(∞, D12, O)
(∞, D21, O)
(∞, D22, O)
① INDICATES COMMODITY NUMBER
(UPPER BOUND, LOWER BOUND, UNIT COST)

2. Many additional constraint the problem more realistic. First am bounds on flows along particular rout

$$x_{ijk} \leq Q_{ijk}$$

where Q_{ijk} is the upper bound on flo upper bound might reflect a capacity route.

The most general form of the t solved using the out-of-kilter algori an index relating to a set of interme sources nor sinks of commodities. Th commodity truck assignment problem.

Minimize: $$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} b_{ijk} x_{ijk}$$

$$\sum_{a=1}^{f} \sum_{j=1}^{n} \sum_{k=1}^{p} b^{**}_{ajk} x^{**}_{ajk}$$

subject to: $$\sum_{j=1}^{m} x_{ijk} + \sum_{a=1}^{f} x^{*}_{iak} \leq$$

$$\sum_{j=1}^{n} x^{*}_{ajk} \leq V_{ak}$$

$$D_{jk} \leq \sum_{a=1}^{f} x^{*}_{ajk} + \sum_{i=1}^{m}$$

$$\sum_{i=1}^{m} x^{**}_{iak} = \sum_{j=1}^{n} x^{*}_{sjk} \qquad \begin{array}{l} a=1, 2, \ldots., f \\ k=1, 2, \ldots., p \end{array}$$

$$\sum_{k=1}^{p} \sum_{i=1}^{m} x_{ijk} + \sum_{k=1}^{p} \sum_{a=1}^{f} x^{*}_{ajk} = \sum_{i=1}^{m} x_{ji}$$

$$j=1, 2, \ldots., n$$

$$\sum_{k=1}^{p} \sum_{j=1}^{n} x_{ijk} + \sum_{k=1}^{p} \sum_{a=1}^{f} x^{**}_{iak} = \sum_{j=1}^{n} x_{ji}$$

$$i=1, 2, \ldots., m$$

$$x_{ijk} \leq Q_{ijk} \qquad \begin{array}{l} i=1, 2, \ldots., m \\ j=1, 2, \ldots., n \\ k=1, 2, \ldots., p \end{array}$$

$$x^{**}_{iak} \leq Q^{**}_{iak} \qquad \begin{array}{l} i=1, 2, \ldots., m \\ a=1, 2, \ldots., f \\ k=1, 2, \ldots., p \end{array}$$

$$x^{*}_{ajk} \leq Q^{*}_{ajk} \qquad \begin{array}{l} a=1, 2, \ldots., f \\ j=1, 2, \ldots., n \\ k=1, 2, \ldots., p \end{array}$$

$$x_{ji} \leq Q_{ji} \qquad \begin{array}{l} j=1, 2, \ldots., n \\ i=1, 2, \ldots., m \end{array}$$

x_{ijk}, x^{*}_{ajk}, x^{**}_{iak}, x_{ji} are non-negative integers

where:

- a = index relating to intermediate nodes
- i = index relating to supply points
- j = index relating to demand points
- k = index relating to commodities
- x_{ijk} = number of truck loads of commodity k sent direc
  from source i to demand j

x^{**}_{iak} = number of truck loads of commodity k sent from source i to intermediate point a

x^{*}_{ajk} = number of truck loads of commodity k sent from intermediate point a to demand j

x_{ji} = number of empty trucks returned from demand j directly to source i

b_{ijk} = unit cost of supplying demand for commodity k at sink j directly from source i = $c_{ijk} + t_{ik} + r_{jk}$

b^{*}_{iak} = unit cost of shipping to a as an intermediate point for commodity k from source k = $c^{*}_{iak} + t_{ik}$

b^{**}_{ajk} = unit cost of using a as an intermediate point for supplying demand for k at j = $c^{**}_{ajk} + u_{ak} + r_{jk}$

c_{ijk} = cost per truck load of commodity k from i directly to j

c^{**}_{iak} = cost per truck load of commodity k from i to a

c^{*}_{ajk} = cost per truck load of commodity k from a to j

c_{ji} = cost per empty truck from j to i

t_{ik} = cost per truck load of shipping commodity k from source i

r_{jk} = cost per tuuck load of receiving commodity k at demand j

u_{ak} = cost per truck load of trans-shipping commodity k at intermediate point a

S_{ik} = supply in truck loads of commodity k at source i

D_{jk} = demand in truck loads for commodity k at demand j

Q_{ijk} = upper bound on flow of k from i to j

Q^{**}_{iak} = upper bound on flow of k from i to a

Q^{*}_{ajk} = upper bound on flow of k from a to j

Q_{ji} = upper bound on flow of k from j to i

V_{ak} = upper bound on trans-shipment of commodity k at intermediate point a

Inequalities (3-16), (3-17) and (3-18) express the constra[ints] on (respectively) the amount of each commodity shipped from i, tr[ans]shipped through a, and received by j. Equations (3-19), (3-20) [and] (3-21) express the conservation of flow constraints that the numb[er] of truck loads that leave a point must be equal to the number of trucks that enter the point. Inequalities (3-22), (3-23), (3-24) and (3-25) restrict the flow between points to be less than a bou[nd]. The integer requirement (3-25a) is satisfied by the solution of t[he] problem as an out-of-kilter graph.

Table 3-1 shows some typical running times for the out-of-kilter solution using randomly generated data.

Table 3-1. Computation Time for Solving Some Multicommodity Tran[s]-Shipment Problems Using the Out-of-Kilter Algorithm*

Number of Sources	Number of Int. Nodes	Number of Sinks	Number of Commodities	Number of Arcs	Number of Nodes	Execution** Time (minutes)
5	5	5	2	205	42	0.04
10	3	3	3	285	59	0.08
15	5	2	2	304	56	0.08
7	4	4	3	288	59	0.08

*Using IBM Share Code 3536 - "Out-of-Kilter or Transportation Problem Solver" and an IBM 7094 computer.

**Includes time to generate graph and costs.

n Algorithm for Constructing a Graph to Represent the Truck ssignment Problem

or each source i, a set of p+1 nodes is created and the nodes re named y_i, y_{i1}, ..., y_{ip}, where y_{ik} represents the k'th ommodity produced at the aource i, and y_i represents the 'th source.

irected arcs are drawn from y_i to each of the y_{ik}. The lower nd upper bounds on capacity of the arcs are zero and S_{ik}^{u}. he unit cost is t_{ik}.

or each intermediate point a, a set of capacitated p nodes $_a = (a^1, a^2, ..., a^p)$ is conatructed. The upper and lower ounds on flow through the capacitated nodes are V_{ak} and zero. he unit cost ia u_{ak}.

or each demand j, a aet of p+1 nodes ia created and the nodes re named q_j, q_{j1}, ..., q_{jp}, where q_{jk} reprcsents the k'th ommodity received at demand j, and q_j represents the j'th emand. Directed arca are drawn from each node q_{jk} to q_j. The ower and upper bounds on each arc are zero and D_{jk}. The unit oat is r_{jk}.

irected arcs are drawn from each node y_{ik} to each capacitated ode z_a, with upper and lower bounds Q_{iak} and zero, and unit ost C_{iak}.

irected arca are drawn from each node y_{ik} to each node q_{jk}, ith upper and lower bounda Q_{ijk} and zero, and unit cost C_{ijk}.

7. Directed arcs are drawn from each capacitated node z_a to ea node q_{jk}, with upper and lower bounds Q_{ajk} and zero, and un cost C_{ajk}.

8. Directed arcs are drawn from each node q_j to each node y_i upper and lower bounds Q_{ji} and zero, and unit cost C_{ji}.

Decoding the Out-of-Kilter Solution of the Graph to Obtain Routings

No fessible solution for the graph means no feasible solution for the problem.

The flow from node y_i to node y_{ik} represents the number of truck loads of k supplied at i.

The flow from node y_{ik} to z_a represents the number of truck loads of k shipped from i to a.

The flow from node y_{ik} to q_{jk} represents the number of truck loads of k shipped from i to j.

The flow from node z_a to q_{jk} representa the number of truck loads of k shipped from a to j.

The flow from node q_j to node y_i represents the number of empty trucks returned from j to i.

The cost of the solution to the graph is the cost of the solution to the delivery problem and is optimal.

collection services. In this chapter,
vehicle capacity is greater than an ir
it is also greater than the sum of al]
all demand locations before returning
problems are referred to as single rot
route need be faahioned. If the sum c
collection vehicle must return to its
completed, and then return to service
becomes a multi-route problem.

Referring to demand as discrete
the context of a network framework. I
located at the nodes of a network with
taking place along arcs. Continuous c
spread along the arcs of the network a
by having the collection vehicle move
of a discrete case might be routing p
petroleum products to gas stations, wh
illustrated by snow removal on streets
Uniform demand means that the demand :
the same as any other point such as
with reasonable accuracy in residen
uniform demand could be exempli:
which could require different quant

Turning to classifications of
problems fall into an area of schedul-

CHAPTER IV. VEHICLE SCHEDULING FROM GIVEN LOCATIONS

Introduction

The previous chapters have been devoted to large scale "f
of goods" problems with the routing of individual vehicles among
collection tasks neglected. This chspter will deal with the pro
of routing of individual tasks through networks for the case whe
the capacity of the vehicle is such that it may service more tha
one demand before returning to its terminal. The problems of co
lecting from a demand area and delivering to it are analogous an
will be mentioned interchangeably. Such operations are the basi
service provided by a multitude of agencies in the private and p
sectors of the economy, and have attracted a great deal of analy
attention.

Distinct areas of application and solution in vehicle scheduling may be classified on the following criteria:

a. What is the capacity of the collection vehicle in relation to the demands for service?

b. Is demand continuous or discrete?

c. Is the demand for service uniform or nonuniform?

d. Are there any additional constraints besides requi
that all demand for service be satisfied by a coll
tion vehicle that starts from and returns to a ter

The first main division may be made on the relationship b
the capacity of the collection vehicle and the size of the deman

ion services. In this chapter, it is assumed that the service capacity is greater than an individual service demand. If lso greater than the sum of all demand, the vehicle may visit and locations before returning to its terminal. These s are referred to as single route problems, since only one eed be fashioned. If the sum of demand is such that the ion vehicle must return to its terminal before collection is ed, and then return to service additional tasks, the problem a multi-route problem.

Referring to demand as diacrete or continuous is done within text of a network framework. Discrete demand implies demand at the nodes of a network with travel between the nodes place along arcs. Continuous demand implies that demand is along the arcs of the network and is satisfied for any arc ng the collection vehicle move along that arc. An example screte case might be routing problems for the delivery of um products to gas stations, while the continuous case is ated by snow removal on atreets or the delivery of mail. demand means that the demand for service at any point is as any other point such as would occur in snowplowing or sonable accuracy in residential solid waste collection. Non-demand could be exemplified by the service stations, each of ould require different quantities of product.

Turning to claasifications of problems, discrete, uniform s fall into an area of scheduling theory known as traveling

salesman problems, and research on these problems has been v

In the traveling salesman problem, a route or schedule must

up which includes all demand locations once and returns to

ing point while minimizing the distance traveled. The tern

traveling salesman problem is generally applied only to the

route problem, while multi-route problems are called m sale

traveling salesman problems, where m is the number of route

fashioned. Since demand is uniform, a knowledge of the capa

the salesman or service vehicle and the number of demand po

be visited, uniquely determines the number of routes.

l this problem to consider not only that all demands must be but that each demand must be serviced within a particular riod or in a certain order. With the addition of such con-, the single route problem as well as the multi-route may no longer be approached directly as traveling salesman . Most of the work done for the case without additional ints is adaptable with modification to this more difficult

If the nature of the demand is uniform and continuous, some ting techniques from graph theory may be brought to bear. gle route problems, the chinese postman problem describes the of traveling all arcs of a network at least once in a con- tour and minimizing the total distance traveled. The problem more difficult if some of the arca in the network are directed) rather than nondirected (two-way). No evidence of work on oute problems in this area has been found, although it would to have great application for many municipal services.

Turning to the solid waste collection problem, it is obvious cause of the aize and complexity of demand it is necessary to th multi-route problems. Arguments may be made as to whether nal constraints are applicable. In the next section, a ure search of work done in all these areas is presented to feeling for the state of the art to this time. Original work author of this dissertation on some problem areas in m sales- blems is also presented.

A. Literature Review

1. The Traveling Salesman Problem. This is the name given to the following problem: A salesman is to visit a group of N ci[ties] and the distances from each city to every other city are known. [Find] the tour that the salesman should follow that will allow him to v[isit] all the cities and return to his starting point while minimizing [the] total distance traveled.

Implicit in the problem formulation is the assumption that this is a single route problem. This means that the capacity of [the] salesman in terms of the number of cities he may visit without returning to the starting point is greater than, or equal to N. If this is not true, the problem becomes a multi-route problem and i[s] called the m salesman problem which is defined in the following manner: A terminal from which a salesman starts and finishes his tours, and a group of N cities with known intercity distances are given. Assume that the salesman can only visit k cities (k less [than] N) at one time before he must return to the terminal. Find the m tours that the salesman must make so that he may visit all the ci[ties] and minimize the distance traveled. Each tour must be of exactly length k plus the terminal, so m is uniquely determined as N divi[ded] by k. If the result of the division is non-integer, dummy cities must be added or real cities dropped in order to make m integer.

By assuming that the salesman must visit exactly k cities on each tour, all considerations of the demand at that city have bee[n]

nd the possibility of making more than m tours by
han k cities on a tour has been ignored.

erature is full of research on the traveling salesman
 it contains the elements of so many scheduling
h diverse areas as job shop acheduling, aircraft
e routing and production planning, but little has
out the m salesman problem. Since most solid waste
lems are multi-route problems, particulsr attention
 this section to those authors' work on the traveling
m that might be extended to the m salesman problem.

eling salesman problem, while being rather simple to
been quite difficult to solve. This is because of
r of feasible solutions that exist for even a moderate
Consider a 20-city traveling salesman problem. For
[1] distance matrix there are 19 factorial possible
0 cities may be ordered on the route. This means
1.2165 x 10^{17} possible feasible solutions, which can
as being a very large number by noting that if one
computer to enumerate one million of these solutions
would take over 27,500 years to search all feasible
s attention has turned towards analytic methods for
ns to the problem. A definitive and complete review
ical development and computational experience with

---

etric distance matrix, the distance from city i to city
essarily the same as the distance from city j to city i.

the trsveling salesman problem wss presented by Bellmore a

Nemhauser (1968). They included a complete bibliography s

cussion of the literature devoted to the subject. Those i

in following the development of the traveling salesman pro

greater detail are directed to that article. In discussin

approsches to the problem, Bellmore and Nemhauser distingu

between three basic procedures. These are tour to tour im

tour building, and subtour elimination. So far, all tour

improvement methods are heuristics[2/] in which rearrangement

ordering of cities on the tour are made until the analyst

fied that the ordering obtained is a good solution. At al

the tour under consideration is feasible in that each city

only once. Switches in ordering of cities that will impro

tour are made, and termination occurs when it appears that

time has been spent searching or no other improvement seem

Of course, such methods do not guarantee optimal results,

do hsve the feature of being quite fast.

Tour building involves choosing a starting city and

electing which city to visit next. The procedure terminat

tour returning to the original city and including all othe

[2/] In discussing these methods, the terms heuristic solutio optimal solution will be used. A heuristic solution is is feasible and that is found by using a set of rules th that it is a "good" solution but there is no assurance t solution found is the best, or optimal. Optimal solutio thst the procedure, if followed to completion, guarantee the answer found is a best solution.

been generated. An example of a heuristic method using this tech-nique is the nearest city rule, which uses as a criterion for selecting the next city to be visited, the unvisited city which i closest. Tour building has also been used in optimal procedures, such as one developed by Little, Murty, Sweeny and Karel (1963). particular interest is a technique developed by Pierce and Hatfie (1966) to deal with a production sequencing problem which has add nal constraints dealing with job deadlines. The algorithm is bas on solving the traveling salesman problem using a tour building optimization technique. Reference is made later in this literatu review to extensions of Pierce's work to single and multi-route truck dispatching problems. Held and Karp (1962) developed a dynamic programming algorithm for the optimal solution of the traveling salesman problem that is limited to about 15 cities by storage requirements on present day computers. Some aspects of Held and Karp's tour building scheme are also applicable to tour building for the m salesman problem but the matter has not been pursued in great detail. Such tour building techniques also alwa work with feasible solutions. This is not true for subtour elimi tion methods, where a less constrained problem is solved which ma or may not give feasible solutions. If the solution to the trave salesman problem is feasible, it is also optimal. However, it ma be infeasible in the sense that all cities are visited but severa disconnected subtours, instead of one tour, are found. The solut is then sought by adding constraints to force the elimination of subtours.

A later section of this dissertation presents a sub
elimination scheme for approaching optimal solution of the
salesman problem.

<u>2. Single and Multi-Route Truck Dispatching Problems</u>. As
the preceding section, the problem to be dealt with here h
at various times referred to as the truck dispatching prob
vehicle scheduling problem, the delivery problem, the clov
problem, the farmer's daughter problem and the truck routi
Briefly, it deals with the scheduling of vehicles of a giv
among discrete, nonuniform demands for the services afford
vehicles. Since demand is nonuniform, the number of route
fashioned is one of the unknowns. In addition to constrai
demand for service must be met, additional constraints on
deliveries, earliest and latest arrival times and vehicle
are also added by some authors.

The earliest description of the problem occurs in a
Garvin et al. (1957). In an article on operationa researc
various aspects of the oil refinery industry, the authors
the large scale problems discussed in Chapter III, and als
the problem of routing of individual vehicles. Motivated
similarity between the vehicle scheduling problem and the
salesman problem, they named the problem the farmer's daug
problem, since the salesman visited certain nodes more tha
that presumably was where the daughter was. No attempt at
tion or solution was reported.

Dantzig and Ramser (1959) discussed a problem they called t ruck dispatching problem. The problem they defined is as follows iven N points or cities each with a demand for deliveries of q_i, nd a terminal point that does not have a demand. Let C = the apacity of the vehicle, and in this problem

$$\max \quad q_i < C < \sum_{i=1}^{N} q_i$$

urther, a distance matrix $D=d_{ij}$ which shows the distance from any ity i, or the terminal to any city, is known. The matrix is sym- etric (i.e., $d_{ij}=d_{ji}$). Find the routing for the vehicle that wil atisfy the demand for deliveries and the capacity constraint on he vehicle while minimizing the distance traveled.

The authors were unable to provide a model formulation that ould solve the problem optimally, but presented an interesting euristic which forms the basis for solution of large scale truck ispatching problems to this day. This is done by investigating t roblem where it is desired to aggregate the various demands into airs. This means that for every city i, the vehicle must make ither the trip from the terminal to the city or the return trip rom the city to the terminal. Since the distance matrix is symme he distance traveled by the vehicle between the terminal p, and t ities is a constant found by calculating

$$\sum_{i=1}^{N} d_{si}.$$

he problem of minimizing the total distance traveled then reduces

to the problem of minimizing only the distance traveled between cities. This may be thought of as finding the pairing of cities that minimizes the distance between cities of each pair.

Define $x_{ij} = 1$ if city i and city j are paired

$= 0$ otherwise

Then by solving

$$\text{Minimize} \quad \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij} x_{ij} \tag{4-1}$$

$$\text{subject to:} \quad \sum_{i=1}^{N} x_{ij} = 1 \qquad j=1, 2, \ldots, N \tag{4-2}$$

$$x_{ij} = x_{ji} \qquad \begin{matrix} i=1, 2, \ldots, N \\ j=1, 2, \ldots, N \end{matrix} \tag{4-3}$$

$$x_{ij} = (0, 1) \qquad \begin{matrix} i=1, 2, \ldots, N \\ j=1, 2, \ldots, N \end{matrix} \tag{4-3}$$

The set of resulting x_{ij} variables gives the desired pairs and consequently the routing for the two city problem. In order to conserve time to solve the integer programming problem, the authors suggest solving the problem with constraint (4-3a) replaced by $0 \leq x_{ij} \leq$ which may yield fractional solutions in some cases. In that case solution is rounded to an integer solution. The problem has been called the matching problem, and may be solved optimally very quickly using an algorithm by Edmonds (1965). The authors' solution for larger problems is to consider the pairs formed at this iteration a single point for the next iteration. In this manner routes are built up until the capacity of the vehicle is met. At each stage

thors used a simple exchange technique to switch pairings if
provement might be found. However, once two cities have been
at one stage of aggregation, they will never be separated at
re level of investigation. The authors noted that the method
ristic, and presented computational experience for a 12 city
n for which the method does not find what they suspected to
optimal solution.

Clarke and Wright (1964) presented a heuristic method based
tzig and Ramser's technique, but one which will allow vehicles
ifferent capacities. Their method varies in the means for
erating the subparts of the problam and in the means for
ing for possible switches at each level. Using the procedure
eveloped, they found a better feaaible solution to the Dantzig
mser trial problem but could not prove that it is optimal.
work serves as the basis for the main computerized vehicle
ling package available from IBM aa a software service to its
This package, "System/360 Vehicle Scheduling Program", Anon.
, available for running on the IBM series 360 computers has
in parts, a network analysis section and a schedule producing
n. The network analysis is designed to find travel distance
avel times between potential delivery points and to do some
inary sorting to eliminate obviously dominated schedules.
data may be in the form of coordinates or distances and the
on of barriers and congested areas may be specified as well.
gorithm used for finding shortest paths through the network is

ills' (1966) work in the decomposition properties of shortest pat roblems in large networks. The scheduling procedure has addition euristics to allow the inclusion of constraints on such factors a riority ratings, earliest and latest possible starting times, everal commodities carried by one vehicle and different vehicle apacities. To date this program is the best large scale analytic ool available for analysis of vehicle scheduling problema.

Balinski and Quandt (1964) presented a formulation for a roblem they called the delivery problem, which may be used to sol mall multi-route truck dispatching problems. They formulated the roblem as a set covering problem which requires that feasible ingle vehicle schedules be enumerated and then chooses from among hese schedules an optimal set meeting the requirement that all emands be satisfied. For a large problem, this would require tha great many schedules be generated first before the optimization echnique could be applied. However, if some intuition is availab r the problem is highly constrained so that the number of feasib outes may be reduced, the method offers some appeal. The problem tatement is as follows:

Minimize: $$\sum_{j=1}^{n} c_j x_j \qquad (4-$$

ubject to the constraints: $$\sum_{j=1}^{n} A_j x_j = E \qquad (4-$$

$$x_j = (0, 1) \qquad (4-5$$

$_j$ is an m by 1 column vector representing the j'th feasible
route, and whose elements

a_{ij} = 1 if the i'th city is visited on the j'th route

= 0 otherwise,

c_j = cost of the j'th feasible route

x_j = 1 if the j'th route is used

= 0 otherwise

E = an m by 1 column vector of 1's

n = number of feasible routes

m = number of cities to be visited.

easible route is represented as a column vector with a one in h position of the vector if the route visits the i'th city ero if it does not. It is desired to find the set of x_j's nimize cost and satisfy the requirement that each city be exactly once. Running the problem as a linear programming without integer constraints on the variables does not yield integer solutions. The authors suggested obtaining a n using an integer programming cutting plane algorithm. They computational experience with a problem of 15 cities and 388 e routes. The feasible routes were examined for dominance[3/] uced to 270. Optimal solution was obtained in 23 iterations, imilar problem ran 200 iterations without solution.

---

oute dominates another if both deliver to the same cities ne has a lower cost than the other.

The idea of using a two phase algorithm in which the firs phase involves generating a set of feasible solutions and the se phase is used to select from among that set the optimal routing quite intriguing. Some form of branch and bound or selective se of the feasible solutions will allow the discarding of many domi feasible solutions before the second phase begins. As constrain on the charcter of the routes become more demanding, the process searching for feasible routes in phase one may even be speeded u since many branches may be terminated early and fewer feasible s tions become candidates for phase two. What is needed is a bett algorithm for solving the set covering problems of phase two, ra than cutting plane algorithms, and the work of Pierce provides a substantial step in this direction.

Pierce has written on several aspects of scheduling and t dispatching, as well as on the development of combinatorial prog ming algorithms for solving set covering problems. As noted ear Pierce and Hatfield (1966) made use of a traveling salesman prob to look at production problems with additional time constraints. This led to Pierce's (1967) major work on truck dispatching. In first part of this work the main concern is with single route pr with additional constraints that include earliest and latest arr times at a specific point, optional deliveries, split deliveries constraints on the service vehicles such as volume limit, maximu number of stops and maximum time per trip. The solution methods suggested by Pierce do not use a set covering approach but are

and bound tour building techniques. The basic element on ranching takes place is whether or not a particular arc from one demand to another is used in the solution, and ound estimates are found using an assignment procedure to that of Little et al. (1963). Two ways for searching nches are suggested. One is "flooding", in which many s are started at once, and the choice of which branch to gate next depends on which seems the most promising at that The other is to follow one branch to completion (i.e., infeasibility, a new feasible solution, or dominance by an g feasible solution). Then backtracking may take place up nch until it is exhausted and another branch is started.

ne problem is characterized by extremely long computation find and verify optimal solutions, Pierce was more interested techniques that generate feasible solutions early so that the re may be terminated and the best feasible solution used as a ic. For this purpose, he feels that on the average the technique will yield feasible solutions sooner.

In a later paper (Pierce, 1968) the author developed a torial algorithm for aolving general set covering problems se as his trial problems a set of truck dispatching problems. tional results based on using the algorithm for different oblems are shown in Table 4-1.

Pierce also found that in dealing with the largest of his s, problem 13, the first feasible solution was reached in

Table 4-1. Computational Experience on Truck Dispatching of Various Sizes Reported by Pierce (1968)

Problem No.	Problem Size (m*n)*	Solution Time in Seconds*** for D Forms of the Algorithm Alg 1	Alg 2
1	5 x 31	0.050	0.050
2	6 x 62	0.167	0.117
3	8 x 92	0.567	0.200
4	13 x 91	35.167	6.367
5	11 x 231	9.183	1.383
6	11 x 561	12.917	2.867
7	11 x 1023	27.267	14.383
8	11 x 1485	34.950	19.317
9	12 x 298	89.133	3.500
10	12 x 538	131.033	7.117
11	12 x 793	77.767	4.567
12	15 x 575	600.000**	69.483
13	19 x 1159	---	2400.000**

*m = number of cities, n = number of feasible routes

**Termination without proving optimality

***Using the IBM 7094

econds, and the best feasible solution at termination had
und quite early (in 349.767 seconds). In a new work on
ments to the combinatorial algorithm, for which only an
t has been circulated, Pierce and Lasky (1969) indicate that
ve found an optimal solution to problem 13 in one-half
This is a considerable time saving, and makes an optimal
re quite competitive with the heuristic solutions for
e size problems for the first time.

Pierce (1969) is continuing work on aolving multi-route
s both by tour building branch and bound algorithms, and by
se, set covering methods. Garfinkel and Nemhauser (1969)
rked on a problem of political redistricting using a two
set covering approach as well.

Andrew and Hamann (1967) also suggested a tour generating
and bound technique. Their work is based on the work of
et al. (1963), and they branch on whether a particular arc is
d or excluded from the tour. A single source for starting
icles, unequal demands at cities, and route capacities (i.e.,
capacities) that do not have to be equal, are features of
el formulation. A bound on each branch is computed by
ng the remainder of the possible arcs which have not yet been
d upon. If a route exists which is below capacity but to
o additional load may be added, a branch must be put in back
starting point. If some branches exist that could possibly
oute over capacity, they are eliminated as well. Then a

ower bound may be estimated by using an assignment procedure wh akes branch assignments based on minimum cost, but may produce ubtours.

The authors reported a great deal of difficulty in conver ence to optimality. They attempted to solve Dantzig and Ramser rial problem and found the same solution as Clarke and Wright d fter 560 iterations, but could not show that it was optimal. T erminated after 47,000 iterations without having finished the otal implicit enumeration. At 14 seconds per thousand iteratio ver 10 minutes were spent without having proved optimality on a 2 city problem. Since a very good solution was developed early he authors suggested that the procedure be terminated early and he best solution found be used as a heuristic solution.

Hausman and Gilmour (1967) dealt with a complicated heuri ased on solving a traveling aalesman problem. They considered ingle source and multi-period demands. Their formulation is as ollows:

$$\text{inimize} \quad \sum_{j=1}^{n} C_i \sum_{i=1}^{m} Z_{ij} + C_2 D_j \quad x \left\{ \max_{i \in s_j} f_{ij} \right\} \qquad (4-$$

here

$$D_j = \min_{W_{ik}} \sum_{i \in s_j} \sum_{k \in s_j} W_{ik} d_{ik} \qquad (4-$$

ubject to: $\quad Z_{oj} = 1 \qquad j=1,\ 2,\ \ldots.,\ n \qquad (4-$

$$\sum_{j=1}^{n} Z_{ij} = 1 \qquad i=1,\ 2,\ \ldots.,\ m \qquad (4-$$

$$\sum_{i=1}^{m} W_{ik} = 1 \qquad k=1, 2, \ldots, m \qquad (4\text{-}10)$$

$$\sum_{k=1}^{m} W_{ok} = n \qquad (4\text{-}11)$$

$$\sum_{i=1}^{m} W_{io} = n \qquad (4\text{-}12)$$

$$W_{ii} = 0 \qquad i=1, 2, \ldots, m \qquad (4\text{-}13)$$

$$Z_{ij}, W_{ij} \text{ integers}; \quad Z_{ij}, W_{ik} \geqslant 0 \qquad (4\text{-}13a)$$

$$\sum_{n \in s} \; \sum_{g \in j} W_{ng} \geqslant 1 \quad \text{all } s, \text{ all } j \text{ (tour requirements)} \qquad (4\text{-}14)$$

S_j = set of all customers in group j

s = any subset of S_j; $s' = S_j - s$

i = a customer; 0 is the terminal

f_i = frequency of delivery required by customer i

d_{ik} = distance from customer i to customer k

j = a group of customers, or route; $j=1, 2, \ldots, n$

n = number of groups (a decision variable)

C_1 = cost per customer delivery

C_2 = cost per mile of truck travel

Z_{ij} = 1, if customer i assigned to group j

= 0, if not

m = number of customers

W_{ik} = 1, if truck travels directly from custom
to customer k

= 0, if not

D_j = optimal (minimum) distance to serve grou

The problem requires in Equation (4-7) that for ea customers considered the minimum cost route for a vehicle This is done using the traveling salesman algorithm devel Held and Karp (1962). For the formulation only the cost traveling salesman solution and not the actual routing is and the Held and Karp algorithm can produce the cost of quickly even though finding the route itself takes longer Equations (4-8) and (4-9) require that all cuatomers be a group and the terminal be assigned to all groups. Equa and (4-11) state that a customer may be visited from only customer, and the vehicle leaving it may go to only one customer. Sending a vehicle back to one self is prohibi and the assignment of flow from and to the terminal must the number of groups (4-12) and (4-11). Inequality (4-14 requirement that statea that for any partition of the cu there must be at least one trip of the vehicle between th Thia requirement is also used in some traveling salesman to restrict subtours. For an N city problem there must conatraints.

To solve the problem the authors generated possib of the customers, calculated the value of D_j for each gr

not directly on the distance between them, but on some indirect measure of the compactness of the area. A final collection are assignment for a crew per work period does not specify an order which it is to be done. The method used for grouping the small areas into work assignments is Hess and Weaver's (1965) work on political redistricting which is itself a heuristic. The autho used this technique to generate several feasible solutions and looked for local improvements by switching some assignments. T have dealt with a fairly large case of an English city of 150,0 population and showed a $25,000 per year decrease in collection costs for the solution generated over the present method of collection.

3. A Note About the Truck Dispatching Problem. All of the aut who have written about the problem to date characterize it as a easy problem to formulate and a difficult one to solve optimall For this reason, many heuristics have been developed, some of w are based on optimization techniques and some of which are not. Spitzer (1969) in an article directed towards the practitioners scheduling, raised the question of using heuristics to find wor solutions versus the use of optimization techniques to find opt answers. He pointed out that real world problems may often hav advantages over theoretical problems in that they may naturally to some form of partitioning or decompoaition. While the first thing taught in a basic systems course is that the best solutio

stem is not necessarily the best eolution for the system as , such approximations might be made along geographic lines. the exponential increase in computation time as the number es increase, it is much easier to analyze three 10 city s than one 30 city problem. Hayes (1967) suggested that the e of optimization techniques may be to test the heuristics. date, little has been done to show how far off a feasible n generated by a knowledgeable operator of a complex vehicle is from the optimal solution. Pierce (1967) and Andrew and (1967) both suggested that the branch and bound routines they vised to find optimal solutions are so slow that they should as heuristics by stopping the procedure at some specified d using the best feasible solution to date.

Chinese Postman Problem. Turning now to the problems of routes for vehicles in networks where demand for service is and continuous along every arc in the network, some early graph theory paves the way for some interesting analytical hes. In 1763, the famous Euler was given the problem of a parade through streets of his native city of Königsberg it would cross all seven bridges in the city only once. gating the problem, Euler found that because of the configu- of the street network and the location of the bridges, the s impossible, and in the process postulated some general bout finding routes that must cover all arcs of a network. r tour is defined as a tour that is continuous, covers every

arc in a network exactly once and returns to the starting
an undirected network,[4] simple rules exist for checking t
euler tour exists or not. A node is even if the number o
touch it is even. An euler tour exists if all of the nod
If some of the nodes are odd then some arcs will have to
more than once. In a directed network where all the arcs
travel limited to only one direction, similarly an euler
if every node is pseudo-symmetric, or the number of arcs
towards the node is equal to the number of arcs directed
it. Rules for establishing an euler tour as well as more
description of the graph theory involved in the process,
in the textbook by Berge (1966).

Finding the optimal route to be taken when an eule
not exist is called the chinese postman problem. In this
route is to be found that minimizes the distance traveled
more than once. Investigators such as Glover (1967) and
(1965) have both suggested optimal algorithms that have b
successful. Work by Edmonds' associates at the National
Standards (Santone, 1968), has been proposed for a case c
waste truck routing in Columbus, Ohio, but the actual wor
yet been carried out.

---

[4] i.e., a network where travel may be in either directio
arc.

Optimal Solution Method for the m-Salesmsn Trsveling

esman Problem

An algorithm for the solution of the m salesmsn traveling
n problem under some special conditions is presented in thia
. The problem to be solved is the following:

Define:

N = the number of cities to be visited excluding the terminals

s = the number of terminals for salesmen

k = the number of cities a salesman may visit excluding the terminals before returning to a terminal

m = the number of salesmen leaving and entering each terminal (the number of tours to be established from each terminal)

Given, N cities each with the same demand for services. The
 number of cities which a salesman may visit is k. There
erminals from which salesmen may begin their tours, viait
 k cities and return to s terminal which need not be the one
ich the salesman started. Each city ia to be visited only
Find the route for each salesman that aatisfies all of the
onstraints and minimizes the total distance traveled by all
esmen.

To eliminate some of the computational problems it is assumed
ch salesman visits exactly k cities. Thus smk=N, which
 that once s, N and k are known, and N/(kxs) is integer,

then m is uniquely determined. If N/(k*s) is not an inte
adjustments such as adding dummy cities or subtracting re
are made.

In the context of the solid waste collection, this
represents the routing of solid waste collection vehicles
transfer stations through their individual collection tas
traveling salesmen are solid waste collection vehicles, t
terminals are transfer stations and the cities are small
aress, each of which generates the same smount of solid w
time period. The assumption thst a vehicle leaves a term
does not return until it has visited exactly k cities clo
resembles the sctual work rules of collection vehicles wi
city. Lacking sny means of monitoring actual load on the
during the collection process, the driver is usually give
assignment and then lesrns by trial and error at which po
lection should be stopped in order to dump the material.
day's work assignment will consist of several collection
that the assumption that a vehicle does not necessarily r
the transfer stназначение from which the route started is not
as long as the last collection route of the day puts the
at the place from which they started in the morning, in o
pick up their belongings and transportation home. The as
that a truck may visit exactly k cities or collection are
that each collection area contributes 1/k truck loads. I
this load must be characterized as a load at a discrete p

it is fairly uniformly apread along the street network to be
in the collection task. This discrete assumption ia less
a continuous assumption. As noted in the literature
continuous problems are chinese postman problems and no
are known for multi-route chinese postman problems.

important feature of the formulation of the above problem
it will allow more than one terminal from which salesmen
and return. This situation is common to many routing
including solid waste collection. The work of some authors
overing solutions to truck diapatching problems, particularly
and Quandt (1964) and Pierce (1967) might also be readily
to conaider the multi-terminal caae. In their work, a two
ocedure is necessary, with the first phase uaed for genera-
sible routes, and the second phaae used for aolving the set
problem necessary to choose the best set of the generated
routes. Phase one could be adjusted to allow the generation
from multiple terminals and the procedure uaed to aolve
covering problem in phase two, slightly modified so that
rminal requirements might be met as well.

C. A Sketch of the Algorithm for the Solution of the m-S

Traveling Salesman Problem

As described in the preceding section the problem a solution technique is desired is that of finding routes exactly k cities for each of m salesmen operating from ea terminals that will visit N cities exactly once and minim total distance traveled. A mathematical programming form the above problem would indeed be cumbersome with respect of variables and constraints necessary to describe even a problem. However, the mathematical formulation of a simp which considers all the constraints except that on the nu cities each salesman must visit, is capable of fairly str forward exposition. Consider the following:

Problem B.

$$\text{Minimize} \quad \sum_{i=1}^{N} \sum_{j=1}^{N} f_{ij}x_{ij} + \sum_{t=1}^{s} \sum_{c=1}^{N} (d_{ct}x^{*}_{ct} + d^{*}_{tc}x^{**}_{tc})$$

subject to:

$$\sum_{c=1}^{N} x^{*}_{ct} = \sum_{c=1}^{N} x^{**}_{tc} \qquad t=1, 2, \ldots, s$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ij} = \sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ji} \qquad j=1, 2, \ldots, N$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ij} = 1 \qquad j=1, 2, \ldots, N$$

$$\sum_{c=1}^{N} x_{tc}^{**} = m \qquad c=1, 2, \ldots, s \qquad (4\text{-}19)$$

$$x_{ct}^{*}, x_{tc}^{**}, x_{ij} \quad \text{are non-negative integers} \qquad (4\text{-}19a$$

ere

x_{ij} = the number of salesmen who travel from city i to city j

x_{ct}^{*} = the number of salesmen who travel from city c to terminal t

x_{tc}^{**} = the number of salesmen who trsvel from terminal t to city c

f_{ij} = distance from city i to city j $(d_{ii} = \infty)$

d_{ct} = distance from city c to terminal t

d_{tc}^{*} = distance from terminal t to city c

m = number of salesmen dispatched from each terminal

N = number of cities

s = number of terminals

Expression (4-15) is the objective function which is the nimization of the distance traveled by the salesmen. Equations -16) and (4-17) express continuity of flow requirements for each rminal and for each city, respectively. They require that the mber of salesmen entering a terminal or city must equsl the numbe aving it. In Equation (4-18) the requirement that exactly one lesman must visit each city is expressed, while in Equation (4-19 salesmen must visit each terminal.

No explicit constraint has been written on the number of

cities a salesman must visit before returning to a terminal. Den as Problem A, the form of Problem B with the added constraint on number of cities a salesman must visit. If Problem B is solved a the solution is a feasible solution for Problem A as well, then a optimal solution has been found for Problem A. If not, a branch and bound scheme based on solution of Problem B with added constraints will be used to find Problem A.

The constraint set of Problem B has an incidence matrix wi a plus one and a minus one in each column, thus indicating it is network problem. This insures that solutions will be integer and that some form of network algorithm may be used to find solutions in a smaller amount of time than a simplex linear programming approach. The graph is in circulation form, which means that the is continuous flow in the graph from sources to sinks and then ba through directed arcs to the sources. The method of solution wil be the out-of-kilter algorithm, but in order to use it some chang in the form of the problem will be made. Equations (4-18) and (4 express the requirements that flows through nodes representing cities must be exactly one, and through nodes repreaenting termin exactly m. This may be accomplished in a graph by the use of a capacitated node, as described in Chapter II.

Each city is represented by a capacitated node with the up bound u_i, equal to the lower bound l_i, equal to one. Entering t receptor of the capacitated node are two seta of arcs: set A_i represents arcs from other cities to i, and aet A_i^* represents arc

terminals to i. Note that $A_i \cap A_i^* = \Phi$, the empty set.

he transmitter node of the capacitated node are arcs which

rise two sets: B_i is the set of arcs which go to other

om i, and B_i^* is the set of arcs that return to the termi-

i. $B_i \cap B_i^* = \Phi$.

a similar manner the terminals may be represented by a

ed node with upper bound equal to lower bound equal to the

salesmen who must enter and leave a terminal m. The

of the capacitated node receives arcs from all cities and

mitter sends arcs to all cities.

nsider the following sets of arcs:

P = (the set of arcs that makes up the capacitated nodes representing cities)

T = (the set of arcs that makes up the capacitated nodes representing terminals)

problem may be shown in simple circulation form as:

nimize $\sum_{i=1}^{p} C_i X_i$

o: $\sum_{i \in B_{(k)}} X_i = \sum_{j \in A_{(k)}} X_j \qquad k=1, 2, \ldots, a \qquad (4\text{-}20)$

$$0 \leq X_i \leq 1 \qquad i \notin P \cup T \qquad (4\text{-}21)$$

$$X_i = 1 \qquad i \in P \qquad (4\text{-}22)$$

$$X_i = m \qquad i \in T \qquad (4\text{-}23)$$

$$X_i = \text{non-negative integer} \qquad (4\text{-}23a)$$

where X_i = flow in arc i

p = number of arcs

a = number of nodes

C_i = unit cost of using arc i

= 0 if $i \in P$

= 0 if $i \in T$

= f_{ij} if i is an arc joining citie

= d_{ct} if i is an arc joining city and terminal t

= d^*_{tc} if i is an arc joining termi city c (where these symbols defined previously)

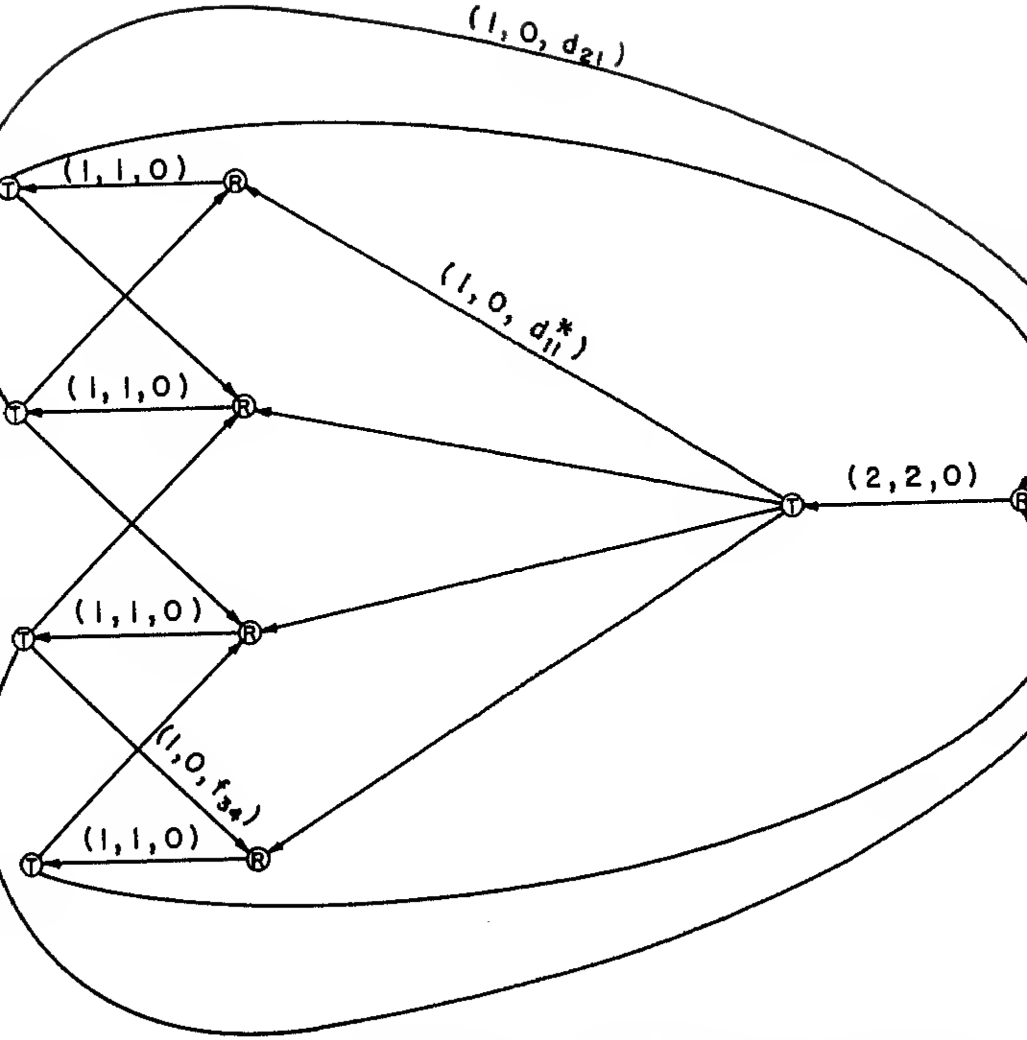
$A_{(k)}$ = arcs leaving node k

$B_{(k)}$ = arcs entering node k

m = number of salesmen

An example of the graph representing this formulati shown in Figure 4-1 for the case of N=4 cities, m=2 salesn terminal, s=1 terminal and k=2 cities to be visited by eac As indicated earlier, such a problem could be aolved by a algorithm instead of the techniques under discussion now. this case is presented here to give an example for a graph easily drawn and understood. The number of arcs and nodes to characterize a problem is quite large. For an N city, problem when all cities may be reached from all other citi terminals, the number of nodes is

$$2s + 2N$$



(UPPER BOUND, LOWER BOUND, UNIT COST
NOT ALL POSSIBLE ARCS ARE SHOWN.

Figure 4-1. Graph for a Problem with N=4, p=1, m=2, k=2.

and the number of arcs is

$$N^2 + 2s*N + s$$

For N=16, s=2 there are 34 nodes and 322 arcs.

The relationship of the solution of Problem B to Pro

may be summarized in the following three cases:

Case 1. The solution of Problem B is also a feasibl solution for Problem A. This is shown in Figure 4-2 This case will be referred to as a feasible solution

Case 2. The solution of Problem B is such that m routes are found from each of the terminals, but not all visit exactly k cities before returning. All cities are visited by a route starting from and re- turning to a terminal. Such a solution is shown in Figure 4-3. This will be called a regular infeasibl solution.

Case 3. The solution of Problem B generates more th ms routes and only ms of these start and end at a te A feasible aolution to Problem B might contain some that start at a city, visit other cities and then re to the starting city without ever touching the termi This will be called a phantom infeasible solution. example is shown in Figure 4-4.

If an infeasible solution such as Case 2 or Case 3 a additional work will have to be done to find the optimal so

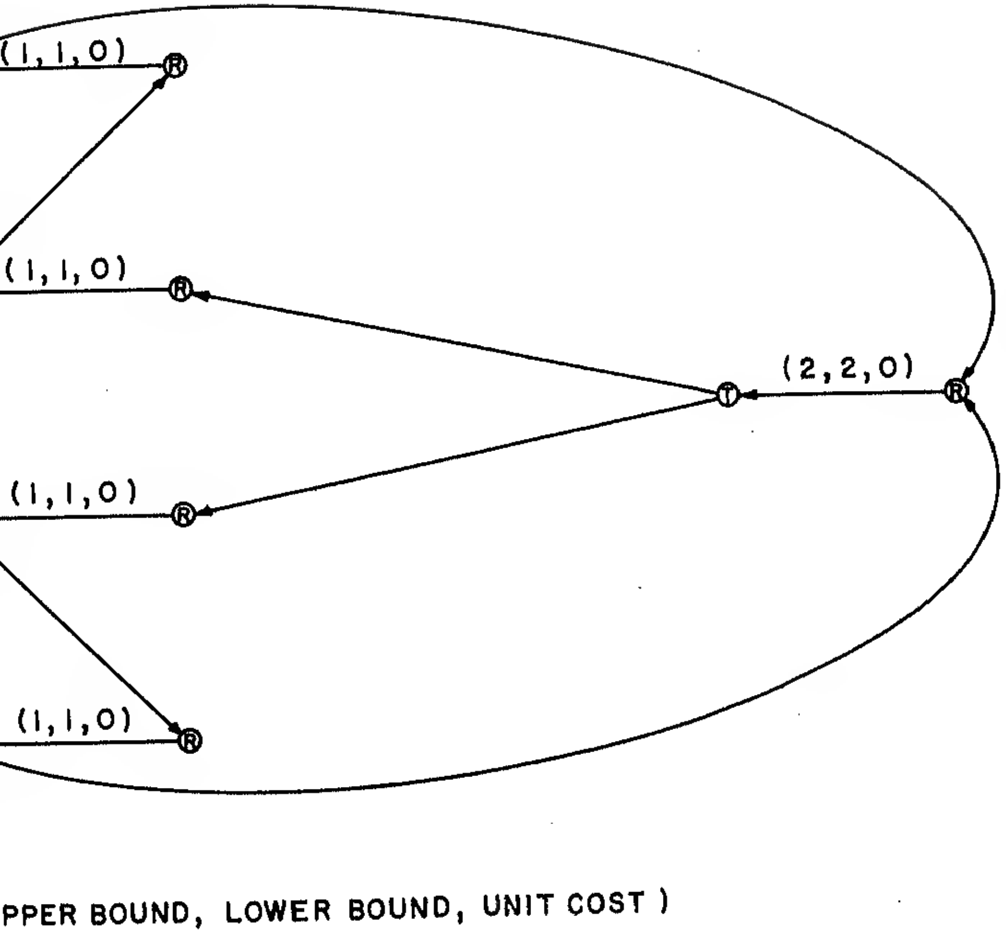


Figure 4-2. Feasible Solution to Problem A.

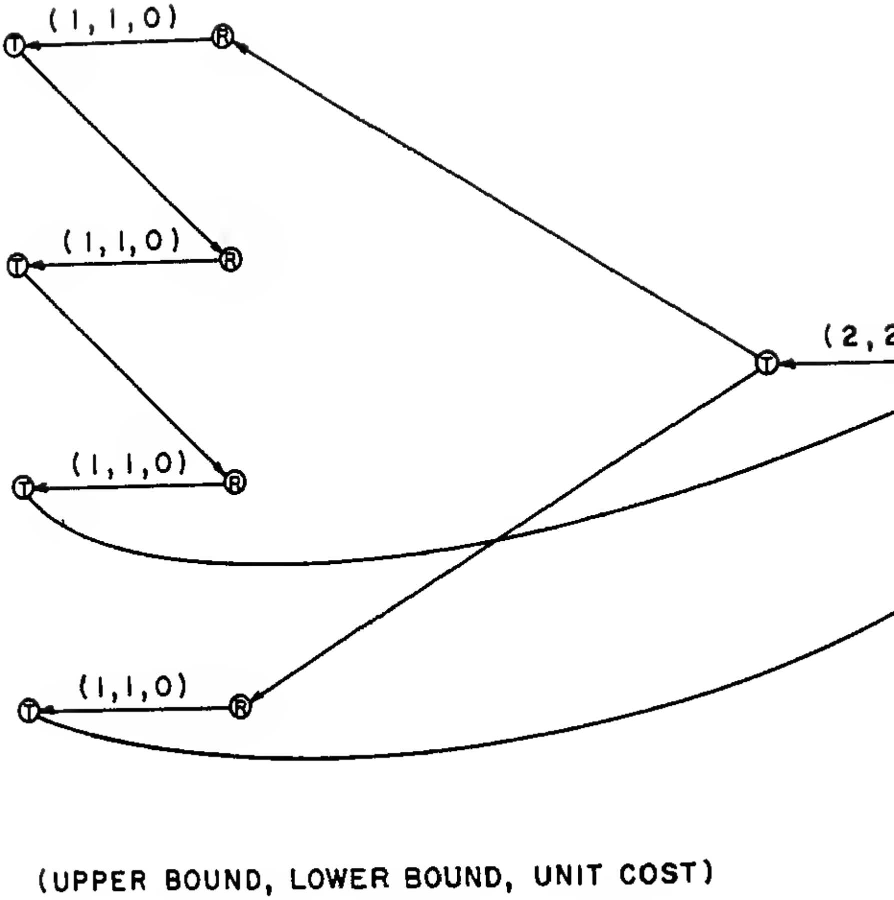


Figure 4-3. Regular Infeasible Solution to Problem

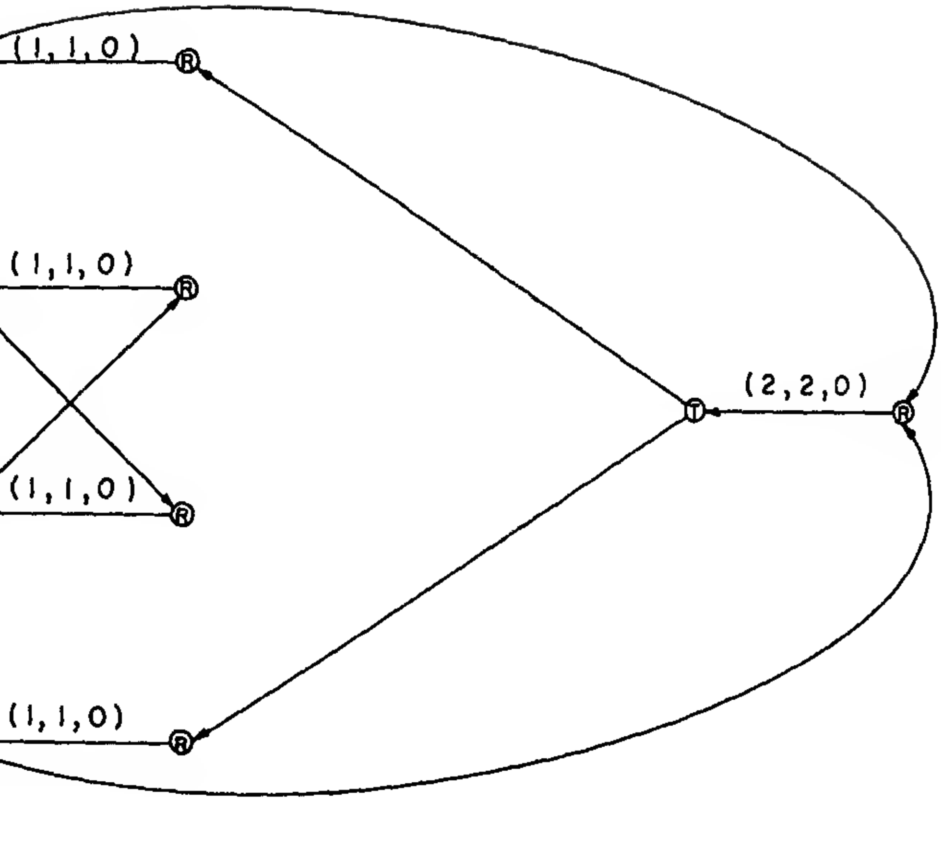


Figure 4-4. Phantom Infeasible Solution to Problem A.

to Problem A. The type of infeasibility will suggest the way tha

the branch and bound procedure can be developed.

The general plan of attack will be the same as for the capacitated trans-shipment facility location algorithm. Solve Problem B. If the solution to Problem B is Case 1, it is feasibl to Problem A. Then since Problem A is more highly constrained th B the optimal solution to A has been found. If the solution to B of Case 2 or Case 3 and therefore infeasible to A, the problem wi be broken into a series of subproblems which divide the set of feasible solutions to B into mutually exclusive collectively exhaustive subsets. This is done by taking some infeasible route the problem solution and prohibiting arcs involved in that route from being in a further solution. Suppose the solution to Proble contains a route which starts at terminal t, visits h cities, $a_1, a_2, \ldots, a_h$ and returns to terminal t^*. Then the set of feasible solutions may be divided into h + 1 subsets with the fir subproblem having all arcs broken from t to all the cities a_1 through a_h. The second subproblem breaks all arcs from a_1 to all other cities (a_1 through a_h) and the terminal t^*. This is contin until the h + 1'st subproblem breaks all arcs from a_h to a_1 throu a_{h-1} and to t^*. The arcs are broken by setting their unit costs infinity. Each subproblem is solved to find its lower bound. Si in each subproblem an arc has been broken which previously had fl into it, the flow pattern will change and the cost of the new solution will be the same or higher than the previous solution.

cost of each of the subproblems is examined and the one with the least cost is found. If its solution is feasible to A, an optim solution to A has been found. If it is not feasible to A, more aubproblems are generated in the same manner and the lowest cost subproblem is again investigated. This is continued until the lowest cost subproblem is a feasible solution to Problem A.

The rules used to set up the subproblems are called break rules. Since there are two different types of infeasible routes it is possible to have several different break rules. One break rule might be to choose for branching the smallest regular infeasible route. Another is to choose for branching the smalle phantom route and break its arcs. Experience with both break rules is reported.

D. A Detailed Algorithm for the m-Salesman Traveling Sales

Problem

1. The following data are given:

   N = the number of cities to be visited excludin the terminals

   s = the number of terminals

   m = the number of salesmen leaving each termina

   k = the number of cities excluding the terminal that a salesman must visit before returning to a terminal

The number of salesmen, m, is an integer determined

If this calculation is not integer, N is adjusted by

dummy cities or subtracting real cities until m is

d_{ct} = distance from city c to terminal t

d^*_{tc} = distance from terminal t to city c

f_{ij} = distance from city i to city j

These distsnces are not necessarily symmetric. Tha

need not be equal to d^*_{ba} nor f_{ij} aqual to f_{ji}.

2. Construct a graph in the following manner:

   There will be two sets of nodes, P and T, which

the graph.

   Define P = (the set of nodes that represent cit

   For each of the N cities, a capacitated node is

lished. This is accomplished by representing the c

node by two nodes with a directed arc between them,

in Figure 4-2. The node that sends the directed arc is known as the receptor node, and the node that receives the directed arc will be known as the transmitting node of the capacitated node. It will be required that the flow throug the capacitated node be exactly one unit. This is accomplished by setting the upper and lower bound on flow throug the arc joining the receptor and transmitting nodes equal t one unit. Refer to the capacitated node representing a cit as a city node.

Define T = (the set of nodes that represent terminals)

For each of the s terminals, a capacitated node is constructed in the same manner as for the cities. It will be required that the flow through each capacitated node representing each terminal be exactly m units, since m salesmen will pass through each terminal. This is accomplished by setting the upper and lower bounds on the flow through the directed arc joining the receptor and transmitting node as m units. Refer to the capacitated node representing each terminal as a terminal node.

Arcs of the graph are defined in the following manner:

<u>From T to P</u>: define a directed arc from the transmitti node of each terminal node to the receptor node of each cit node. The unit cost on the arc is d^*_{tc} and there is an uppe bound of one on the arc.

From P to T: define a directed arc from the trans
node of each city node to the receptor node of each te
node with a unit coat of d_{ct} and an upper bound of one

Within P: from any city node i to any other city
$i \neq j$, define a directed arc with unit coat f_{ij} and an
bound of one, from the transmitting node of i to the
receptor node of j.

3. Define S = (set of all arcs of the graph that hav excluded from the solution and hence have zero flow)

$\underline{S}$ = (set of all arcs of the graph not yet from the solution)

$\underline{S}^*$ = (set of all arcs of the graph)

Then $S \cup \underline{S} = \underline{S}^*$ and $S \cap \underline{S} = \phi$, the empty

Initialize $S = \phi$

$\underline{S} = \underline{S}^*$

4. Find the minimum cost, maximum flow through the gr
uaing the out-of-kilter algorithm. If this is the fi
step 4 has been entered and there is no feaaible solu
to the graph, there is no feasible solution to the pr
Define the cost of this flow as CGRAPH.

5. Examine the solution of the graph to see if it is
feasible solution to the m salesman traveling salesma
This is done by decoding the solution to determine th
for each aalesman. From each terminal node only m ar

leaving it may have non-zero flow and for each city only one arc entering and one arc leaving it may have non-zero flow. The routes for each of the salesmen may then be calculated by starting at a terminal and selecting an arc to a city, say city i^*, from that terminal with non-zero flow. Then examine the city node i^*, and find the single non-zero arc leaving it. If this arc goes to a terminal the route is complete. If it goes to another city, say i^{**}, examine the city node i^{**} and apply the same rules until the route is terminated. Continue until each of the m routes at each of the s terminals has been enumerated. If each of the routes enumerated visits exactly k cities, the solution is feasible.

6. If the solution is feasible, go to step 7. If it is not, one of the infeasible routes is chosen for branching. Select the smallest route in terms of number of cities visited as the candidate for branching. In case of a tie for the smallest, choose the first enumerated.

7. Store information about the solution. File on a list, ranked on the cost CGRAPH, of all solutions generated, data about the feasibility, members of the S set and the candidate for branching.

8. Remove the first solution on the list. If that solution is feasible, it is optimal.

9. If the solution is infeasible, several subproblems
set up. Suppose the route chosen as the candidate for
branching starts at terminal t, visits h cities, a_1, a
..., a_h, and returns to terminal t^*. Then h + 1 subpr
will be generated. First eliminste all arcs listed in
S set by setting their costs to infinity.

9a. Subproblem 1 - Break all arcs from t to each
the cities a_1, a_2, ..., a_h, by setting the costs
these arcs to infinity and add these arcs to the S
Repeat steps 4 through 7. Restore the arcs just b
to their original costs and go to 9b.

9b. Subproblem 2 through h + 1 - If this is the g
subproblem, break all arcs from a_{g-1} to each of th
other h-1 cities and to t^* and add these arcs to t
S set. Repeat steps 4 through 7 and restore the a
just broken and go to the next subproblem. When a
h + 1 subproblems are completed, go to step 8.

tational Experience

e algorithm was programmed in ASA Fortran IV and run on an computer. The out-of-kilter algorithm was written by d made available through the IBM SHARE Library. The list g subroutines used for keeping track of solutions were y Bellmore (1966). Two posaible break rules were considered ng up subproblems. Break rule A selected the smallest e route that passed through a terminal as the route for . Break rule B involved selecting a phantom route (Case 3 lity) which by definition does not pass through a terminal, ndidate for branching. If no phantom routes exist, rule A ed. Both rules were tried in early runs for some 8 city and showed little difference. Only break rule A was used rger city cases. Because of dimension limitations and the cimal packing instead of binary packing for the storage of on about aolutions, many problems were limited by exceeding ather than by time limits. Future attempts at working with rithm will require better programming techniques and more acking of data to insure greater efficiency, but such pro- efforts were not deemed necessary for this trial demon- of the method. Runs were made with one and two terminals of 8, 12 and 16 cities. Results of these runs are shown 4-1. Runs were made with symmetric and asymmetric data. tric case generally took longer to solve. This same

property in traveling salesman problems is explained by Bel and Malone (1968). The procedure has a tendency to produce few feasible solutions before an optimum is found. This wo a considerable disadvantage in the last two runs, since con time was invested before termination without finding one fe solution. The algorithm could be speeded up and have more solutions generated, by taking each infeasible solution and some heuristic procedure to round it to a feasible solution development of the feasible solution would not take long, a found to be lower than the current best feasible solution, bound on the problem has been found. At termination before solution, a good feasible solution would still exist.

Edmonds and Johnson (1969) are presently working on area known as degree constrained subgraphs which would appe have application to the above problem. Basically, the prob to minimize the cost of using arcs subject to constraints t specified number of arcs must enter and leave each node. T form the basis for generating tours for each vehicle. The Edmonds and Johnson deal with are as fast as the matching p algorithms. Based on Bellmore and Malone's (1968) experien would be expected that this would alleviate the problem of different computation time for symmetric and nonsymmetric d cases.

Table 4-2. Computational Experience for Problems with Randomly Generated Data

No. of Terminals (s)	No. of Vehicles at each Terminal (m)	No. of Cities (N)	Data Type	Features of Graph: No. of Arcs	Features of Graph: No. of Nodes	No. of Feasible Sol'ns	No. of Times OKA Called	No. of Feasible Sol'ns Found	Running Time on IBM 7094 (minutes)
1	2	8	ASYM***	73	18	1,680	8	2	0.03
1	4	8	ASYM	73	18	27	27	2	0.10
1	4	12	ASYM	169	26	1,360	76	1	0.48
1	2	12	ASYM	169	26	665,280	150	2	0.75
1	3	12	ASYM	169	26	11,880	101	2	0.52
1	3	12	SYM***	169	26	5,940	101	2	0.52
2	2	12	SYM	169	26	47,520	46	1	0.32
1	4	16	ASYM	289	34	43,680	251	4	1.87
2	4	16	ASYM	322	36	50,000	210	8	2.39
2	2	16	ASYM	322	36	174,720	215	3	1.87*
2	4	16	SYM	322	36	25,000	230	0	2.12**
2	5	20	ASYM	1,682	44	4.4×10^6	330	0	3.21**

*Stopped because of storage with lower bound close to a generated feasible solution.

**Stopped because of storage with no feasible solution found.

***ASYM means asymmetric data.

SYM means symmetric data.

CHAPTER V. THE ANALYSIS OF A SOLID WASTE COLLECTION SYSTEM

Introduction

In this chapter, a large scale solid waste collection an disposal system will be investigated, using some of the techniq suggested in previous chaptera. The aystem chosen for examinat was that operated by the City of Baltimore, Maryland. Thia selection was based on several factora. First, Baltimore is a good example of a large city with extensive investment in a pub solid waste collection syatem. Second, a great deal of data ha collected concerning the operation of the system, and is avail Analysis of the Baltimore system has already been attempted by Truitt (1968, 1969), who used the city as an example in the development of a simulation model of collection. Through his excellent work, considerable data and insight into the system a available for use with the models proposed in this study. All taken from Truitt's work will be referenced.

acterization of the Baltimore Solid Waste Collection System

he City of Baltimore, Maryland, had a population in 1960 of r one million, a land area of 80.3 square miles and physical ristics that ranged from high density urban slums to verdant ity residential subdiviaions. The municipal functions of ste collection and disposal as well as general sanitation ea are carried out by the Bureau of Sanitation of the nt of Public Works. For purposes of logistics and super- the city is subdivided into five autonomous districta, the tern, the northeastern, the central, the eastern and the These districts are shown on the map in Figure 5-1. The mploys 1271[1/] persons and owns approximately 340 vehicles es of equipment, and operates on a budget of over seven dollars per year. A breakdown of the functiona and their shown in Table 5-1. Aa indicated in the table, collection for almost 85 percent of the department's operation and ominant collection operations are mixed refuse collection et cleaning.

he Bureau collects mixed refuse from residential and non- al sites twice a week, once on either Monday, Tuesday or y, and once on either Thursday, Friday or Saturday. Since on in the earlier part of the week represents the pick up days' accumulation of wastes, crew size is one driver and

---

eport of Department of Public Works (Anon, 1966a)

three laborers. In the later part of the week crew size is re to one driver and two laborers. Two types of vehicles are gen used for collection. They are both equipped with compaction e ment and have capacities of 20 and 13 cubic yards of compacted wastes each. The larger vehicle is often referred to as a fiv truck and the smaller as a three ton truck. The larger vehicl thought to be more efficient than the smaller and is assigned general tasks while the smaller is saved for alleys and other restricting jobs. In 1965 the Bureau collected 336,893 tons o mixed refuse, 31,395 cubic yards of ashes and 216,622 cubic ya street dirt. The collection of the mixed refuse from residenc involves 92 vehicles, each with three daily route assignments week for a total of 276 routes. There are two city owned inci tors for the disposal of wastes, and the solid waste collected taken directly from the collection area to the incinerator by collection vehicle. One of the incinerators is the Number Thr (Reedbird) with a capacity of 600 tons per 24 hour day, and th other is the Number Four (Pulaski) with a capacity of 800 tons 24 hour day. The location of these facilities is also shown i Figure 5-1.

-1. Cost of Operations and Services Provided by the Bureau of Sanitation, City of Baltimore, Maryland, 1965.

Operation	Yearly Expense	Percent of Total
et Cleaning	2,792,000	37.2
Cleaning	202,000	2.7
d Refuse Collection	3,034,000	40.4
r Miscellaneous	324,000	4.4
ion Expenses	6,352,000	84.7
nerators	1,110,000	14.8
s	40,000	0.5
l Expenses	1,150,000	15.3
	7,502,000	100.0

B. Study Objectives

Analysts involved in the study of, and planning scale public systems such as the solid waste collectio systems must be concerned with both long range and sho planning. Not only must they study the evolution of n and methods that may dramatically change alternatives in the future, but they must be concerned with the pre vital existence of the system. This study will be dir the analysis of the system within the short range pict to give some feeling for the alternatives that are ava immediate introduction into the system. These alterna physical changes such as the building of new structure changes such as changes in the manner and character of and rules of operation of the system, or changes in th carrying out certain objectives of the system. Short tives that fall into each of these categories make up issues that require analytical attention.

The three questions that make up the theme of t

1. Are transfer sites within the city a feasib If so, where should they be located and to should they be built?

2. What is the cost and effect of increasing c frequency from twice a week to three times

3. Under what conditions and at what price wou haul become a feasible alternative for the

Subordinate to these main questions but still of great

lyst and planner are such additional questions as:

4. How sensitive is the system to changes in parameters and cost estimates?

5. What are the effects of political and esthetic constraints that might force a change from the solution most economically efficient from the engineering point of view to one that is perhaps more acceptable to segments of the community?

6. Are there advantages to cooperation between governmental units within a region where such cooperation does not now exist?

uestions will be subject to some preliminary investigation next sections, using the large scale facility location model ter II. More detailed study would also require some of the techniques of Chapter IV to achieve overall efficiency of on, but these have not been included in this overview.

C. Calculations and Data for the Baltimore Study

1. Collection area to be investigated

The area chosen for study was the same area st Truitt (1968, 1969). It comprises the northwest divis City of Baltimore, and a map of the area and its relati the city are shown in Figure 5-1. Within this area i to aubdivide into smaller subareaa for analysis. A c to do this is to consider each census tract within the subarea. Exact population and neighborhood density t the 1960 census, are accurately known. There is a to census tracts within the northwest quadrant and these the map of Figure 5-3. A coordinate system has been zero point at the northwest corner. Information abou number of housing units, neighborhood density type an coordinatea of the approximate centroid of the tract Table 5-2. Neighborhood density types are based on e from cenaua data by the City of Baltimore, Department as reported by Truitt (1968). These definitions are Table 5-3. The housing unit density is important bec aome indication of the speed of collection within a s would be suspected that the more closely spaced the p are, the faster the collection process may be in term per hour collected. Truitt, in a atatistical study o ata, shows that there is a aignificant difference be

able 5-2. Pertinent Data Concerning the Forty Census Tracts of th Northwestern Division, Baltimore, Maryland

ract umber*	Neighborhood Density	Number of Household Units**	Population** 1960	Coordinates x	y**
8-1B	1	2,482	2,482	3,750	13,00
8-2	1	2,035	5,646	3,806	9,00
8-3	1	1,194	3,546	4,000	26,00
8-1A	2	1,099	3,531	4,000	8,50
7-19	1	1,490	4,622	10,000	6,50
7-20	1	4,634	13,437	5,750	3,00
7-17	2	2,079	6,756	14,500	8,50
5-12	3	2,011	6,091	17,500	17,00
5-13	3	1,910	5,623	15,000	14,50
7-16	3	1,979	6,127	16,000	12,00
7-18	3	3,207	9,808	10,500	11,00
5-7A	2	865	2,944	14,000	22,50
5-8A	2	1,787	4,811	11,500	21,50
5-8B	2	493	1,390	11,500	25,0
5-9	2	1,574	4,950	9,000	24,5
5-10	2	2,249	6,193	9,250	16,5
5-11	2	2,691	7,677	13,500	17,0
7-11	1	780	2,786	31,500	10,5
7-12	1	2,159	6,648	31,000	3,5
7-13	1	913	3,183	25,000	4,0
7-14	1	1,543	4,398	26,500	11,0
7-15	1	1,904	6,427	20,000	6,5
2-1	2	1,887	3,495	32,000	15,0
3-5	3	523	1,762	27,500	20,5
3-6	3	1,426	4,336	27,500	18,0
3-7	3	1,918	5,429	27,500	15,0
3-8	3	2,039	6,824	22,500	15,5
2-2	4	3,839	7,499	32,000	16,5
2-3	4	1,987	5,483	33,000	22,0
2-4	4	1,687	5,469	33,000	24,
12-6	4	1,532	3,177	32,000	22,0
12-7	4	1,474	4,705	30,000	22,
13-1	4	1,917	4,648	27,500	23,
13-2	4	2,062	5,859	27,500	25,
13-3	4	1,598	4,904	23,500	20,
13-4	4	1,154	4,026	22,500	20,
15-4	3	1,576	5,800	20,500	25,
15-5	3	741	2,064	18,000	22,
15-7B	3	1,787	4,298	14,500	25,
15-6	3	2,150	7,725	14,000	27,

*Tract number is U. S. Census designation.

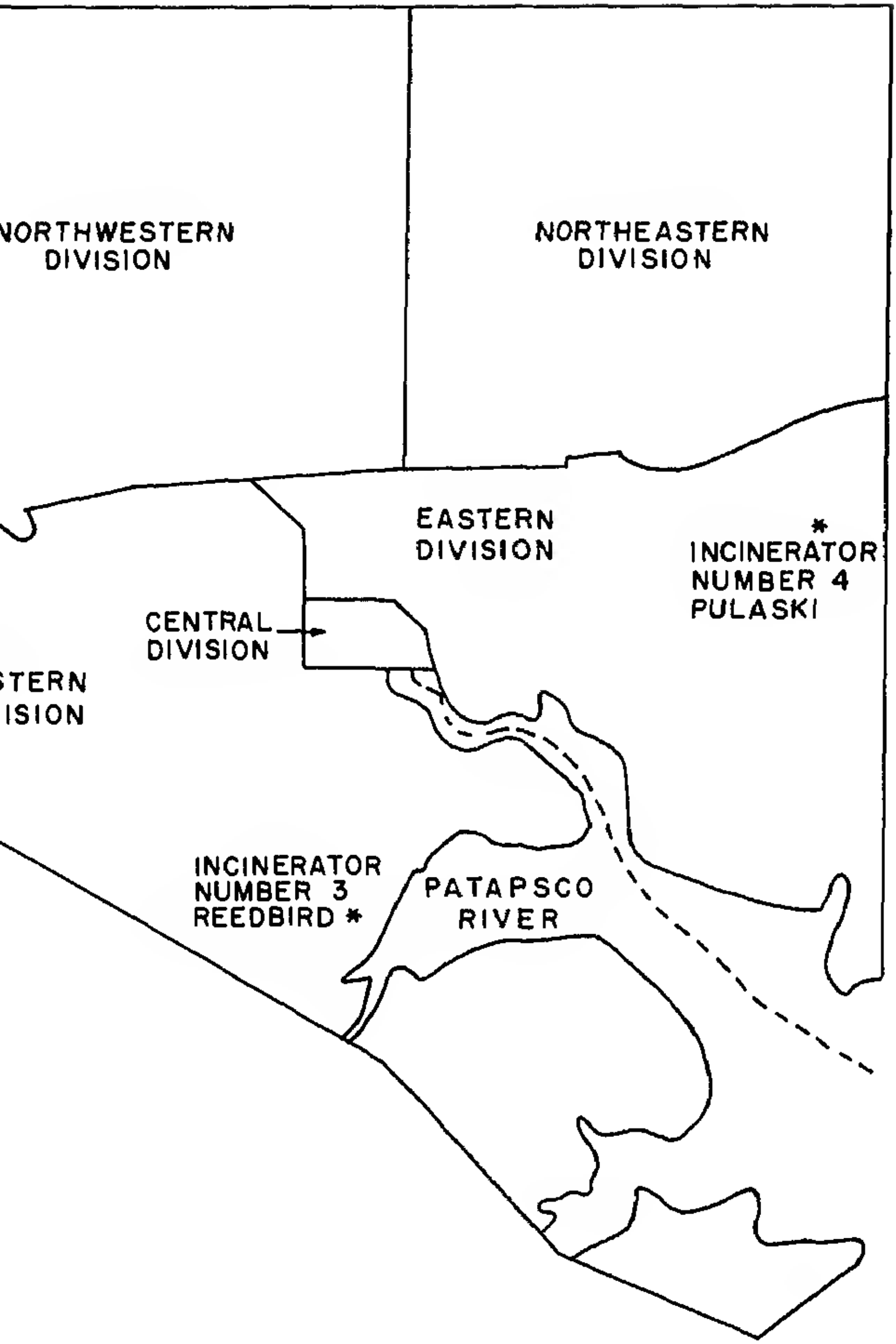
**Population and number of household units from 1960 census.

***Coordinates are in feet measured to the approximate centroid o tract. Point x=0, y=0 is located at the extreme northwest cor of the Baltimore City boundary.

type 1 and density types 2, 3, and 4, but no signific
between 2, 3, and 4. Thus only two sets of neighborh
types need be considered, type 1 and other than type

Table 5-3. Classification of Neighborhood

Classification	Housing Units per
1	Ten or less
2	Eleven to twenty
3	Twenty-one to for
4	More than forty



5-1. Operating Divisions of Bureau of Sanitation, Baltimore, Maryland.

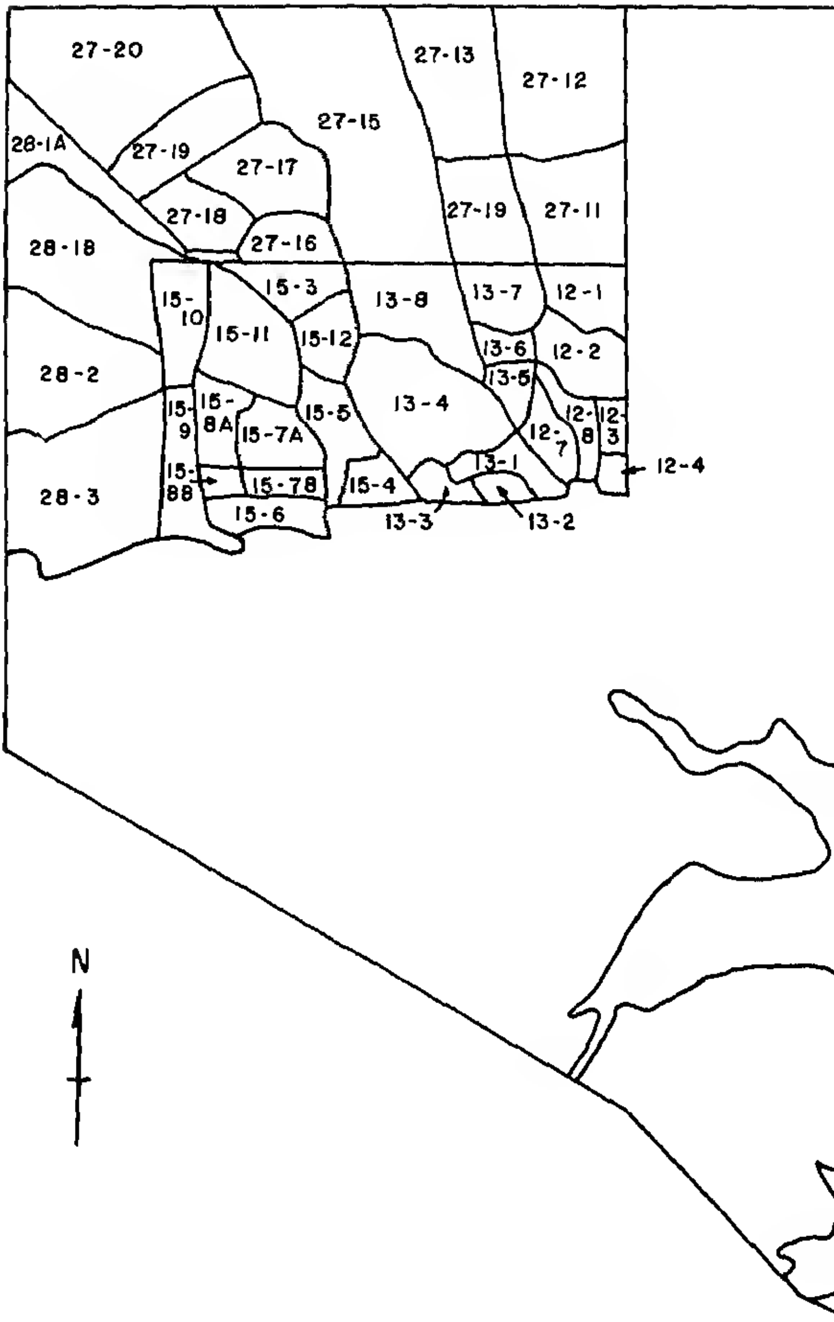


Figure 5-2. Map Showing Census Tracts in Northwe
of Baltimore, Maryland.

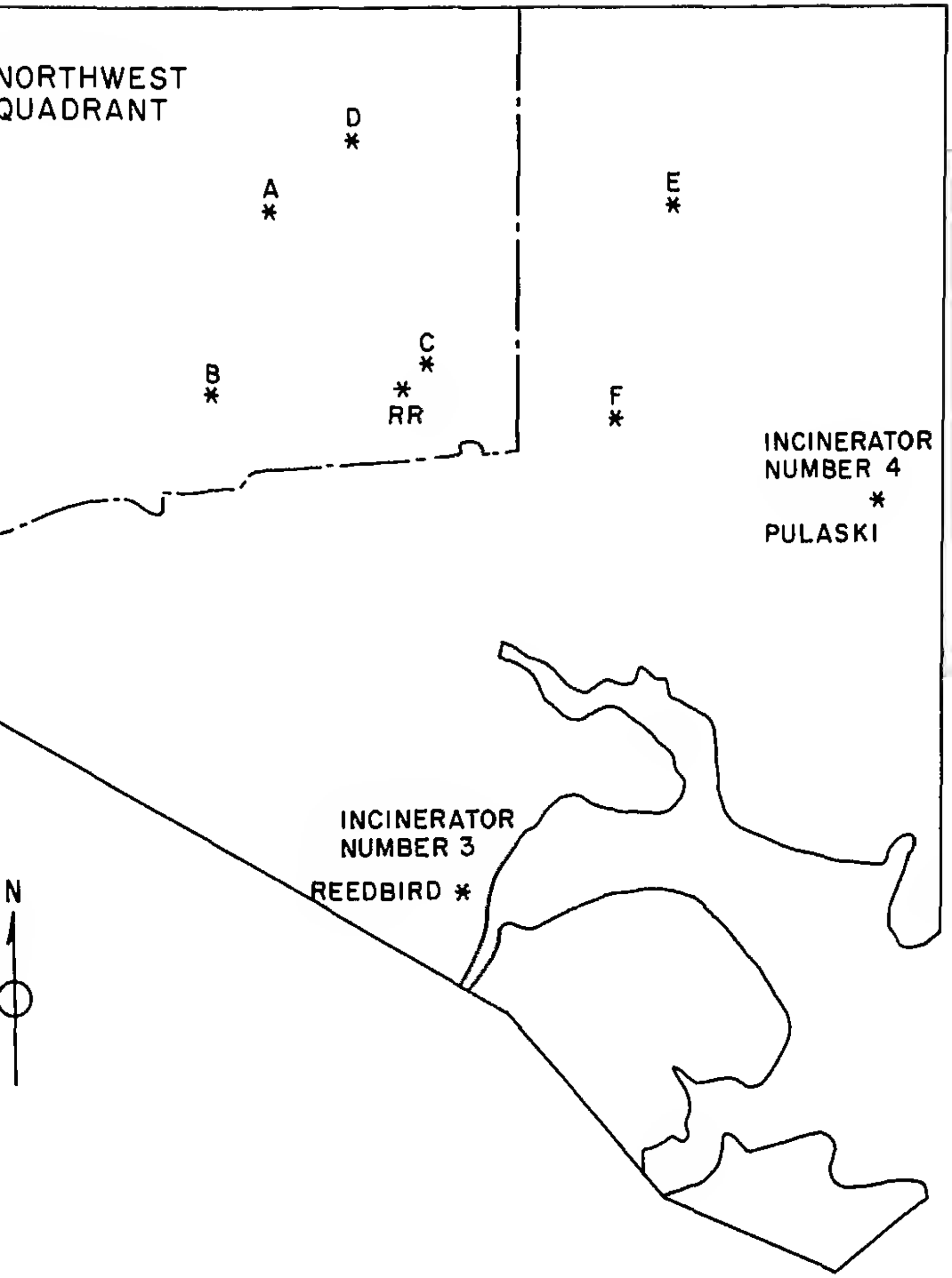


Figure 5-3. Map Showing Location of Proposed Transfer Sites and Present Incinerator Sites, Baltimore, Maryland.

2. Speed of collection within a subarea

The speed of collection within a subarea is a f
neighborhood density and the number of days since the
tion. When the collection frequency is two times a we
Baltimore, one collection ia of four days of solid was
and the other of three. Normally the City of Baltimor
driver plus three laborers for the larger collection,
plus two laborers for the smaller one.

On pages 174-176 of Truitt (1968) several histo
presented for collection rates in pounds per hour as s
days since last collection and neighborhood type. The
based on twice a week collection are computed from act
tions within the system. Estimates were made of histo
times a week collection frequency. In this case the f
of the week has three days accumulation and the remain
two days accumulation. The number of personnel assume
and two laborers. Table 5-4 presents weighted average
grams for different conditions which will be used in t
models.

3. Cost of collection within a subarea

Coat of collection is assumed to be a function
frequency, the speed of collection, the neighborhood t
tion, and the work rules assumed. The following data
to this computation:

-4. Average Collection Rate in Pounds per Hour for Different Days Since Last Collection and Neighborhood Type

nce Last ection	Neighborhood Type	Average Pounds per Hour Collected
2	1	2,500
2	2, 3, 4	4,580
3	1	3,000
3	2, 3, 4	6,300
4	1	5,000
4	2, 3, 4	6,300

Table 5-5. Collection Costs in Dollars per Hour

nce ion	Labor Necessary	Labor Cost Dollars/Hr	Vehicle Cost Dollars/Hr	Total Cost/Hr
	1 driver + 3 laborers	12.63	4.40	16.73
	1 driver + 2 laborers	9.33	4.40	13.73
	1 driver + 2 laborers	9.33	4.40	13.73

a) Collection vehicle costs - The City of Baltim estimates its cost per hour of vehicle use for the 20 cu collection vehicle as $4.40 per hour.

b) Typical labor rates are twenty dollars per da driver and eighteen dollars per day for a laborer. The assignment for a collection of four daya aince last coll one driver and three laborers per vehicle. This drops t and two laborers for shorter intervals between collecti

c) To compute an hourly labor rate from the dail is necessary to compute the average time spent in actual Truitt (1968) has estimated that between 45 and 50 minut are spent in actual productive time, with the remaining spent in start up, repairs, off service time, etc. For of this study, 45 minutes per hour or 6 hours per day wi as the productive time. This yields a productive hourly drivers of $3.33 per hour, and for laborers of $3.00 per

Tsble 5-5 shows labor, vehicle and total costs fo types of collection.

d) Waste load generated - The waste load generat person per day is a statistic that shows great variation (1968) after extensive study of Baltimore residential ds 1.95 pounds per person per day and calculated a standard of 0.09 pounds per day. This figure will be varied to s sensitivity of the solution to it.

e) Total collection cost in a subarea per week -

llection cost per week in a subarea is calculated in the following nner:

Define p_i = population of the i'th subarea

k_i = weight of solid waste generated per person per day with twice a week collection frequency

k_i' = weight of solid waste generated per person per day with three times a week collection frequency

l_j = labor cost per hour for a crew of one driver plus j laborers

g_{ik} = collection rate in pounda per hour for neighborhood type k for i days since last collection

The reasons that the waste generation is shown to be different different types of collection frequency are pointed out in statis al studiea by Quon, Tanaka and Charnes (1968). They show a nificant increase in waste per household per week when collection quency increases from once to twice a week.

Total collection cost in the subarea is C_i.

$$C_i = P_i k_i \left(4\frac{l_3}{g_{4,j}} + 3\frac{l_2}{g_{3,j}}\right)$$ dollars per week for a collection frequency of twice a week, and

$$C_i = p_i k_i' \left(3\frac{l_2}{g_{3,j}} + 2\frac{l_2}{g_{2,j}} + 2\frac{l_2}{g_{2,j}}\right)$$ dollars per week for a collection frequency of three times per week

collection cost per ton = $\frac{C_i}{7p_i k_i} \times 2000.$

extra charges were made for overhead or supervision as there was way to obtain a reliable estimate of these figures.

4. Cost of Transport by Collection Vehicle from t to Transfer Facilities or Disposal Points

a) The speed in traffic of a collection vehicl not to differ significantly between a loaded and an un condition by Truitt (1968), whose histograms also indi average speed of 16 miles per hour.

b) The cost of the vehicle and crew per hour i calculated in 3 (c) above for collection within a sub

c) The distance between the subarea and a disp fer site ia calculated in the following manner. A set coordinates are given for all transfer stations and di The location of the subarea is taken as its centroid. vehicles usually cannot travel in straight lines betwe travel distance is estimated using a metropolitan or r measure in which the travel distance between a and b i

$$|X_a - X_b| + |Y_a - Y_b|$$

where $| \quad |$ meana absolute value.

d) The average weight carried per 20 cubic yar vehicle at the disposal point is indicated in a histog in Truitt (1968). This is taken as 9,000 pounds per v the cost per truck for travel from the disposal point collection area and return is measured in the followin

Define d = travel distance in miles in one direction

l_i = cost of vehicle + driver + i laborers per hour

s = travel speed in miles per hour

hen cost per truck load for the two way trip is

$$\frac{2\ d}{s} \times l_i$$

e each truck carries on the average 9,000 pounds or 4.5 tons,

per ton is

$$\frac{1}{2.25} \times \frac{d}{s} \times l_i$$

e 5-6. X, Y Coordinates for Proposed Transfer Stations*

cation ol	Type	X Coordinate (feet)	Y Coordinate (feet)
	Transfer to railroad cars	24,000	21,000
	Transfer to trailer trucks	16,000	11,000
	Trsnsfer to trailer trucks	14,000	22,000
	Transfer to trailer trucks	28,000	20,500
	Transfer to trailer trucks	22,500	7,000
	Transfer to trailer trucks	37,000	9,500
	Transfer to trailer trucks	34,500	21,000

, Figure 5-3.

5. Transfer Facility Costs

Truitt (1968) listed four potential transfer si located within the northwest quadrant, which will be u study as well. In addition, three more sites were cho quadrant at various locations including one that is in transfer to rail-haul selection. The coordinates X, Y site are given in Table 5-6 and are shown on the map i The original Truitt sites are marked A, B, C, and D. additional vehicle transfer sites are E and F, and the is RR. The cost of building transfer facilities was a by Truitt (1968) in the following manner:

Land Cost: Fixed cost of $40,000 below 200 capacity and a variable cost of of additional capacity above 200

Labor Costs: Three men at $20 per day for a t station of less than 200 tons pe per day. Four men at $20 per da day for a greater capacity.

Utility Cost: $1200 per year.

Capital Cost of Structures: $125,000 for a capacity of less per day. Above 100 tons per day variable cost of $500 per ton of capacity added to the fixed char

The total waste load per week from the northwest quadran estimated as 14 pounds per week for each of the 203,500 persons 1428 tons per week. The peak day load would occur in the early of the week with three times a week collection. In this case o Monday and again on Tuesday, one half of the northwest quadrant visited and 3/7 of the weekly waste load is collected. Thus th maximum daily load is

$$1/2 \times 3/7 \times 1425 = 306 \text{ tons per day}$$

The cost of a transfer facility as a function of weekly capacity is shown in Table 5-7. The yearly cost of the structu has been produced through discounting, assuming 30 year life anc interest rate of 10 percent. The yearly cost of the land is bas on interest on investment and it is assumed that the land does r depreciate. No estimates were made on taxes lost on the land, since Truitt's sitea A, B, C and D were chosen on land that waa already public. It is asaumed for simplicity that the same type of land is available at E and F.

6. <u>Transfer from Transfer Station to Final Disposal Via Transport Vehicles and Disposal Costs</u>

a) Transfer costs - The vehicles suggested for transport the final disposal areas from the transfer station are of two ty One type is a 75 cubic yard tractor trailer with a weight capaci of an estimated 35,000 pounds. Truitt (1968) gives the cost of

Table 5-7. Cost of Transfer Stations as a Function of Weekly Capacity

Weekly Capacity (tons/wk)	Total Structure Cost (dollars)	Total Land Cost (dollars)	Yearly Structure Cost* (dollars/yr)	Yearly Land Costs** (dollars/yr)	Utilities (dollars/yr)	Labor*** (dollars/yr)	Total Cost per Yr	Total Cost per Wk
600	125,000	40,000	13,300	4,000	1,200	18,720	37,220	719
900	150,000	40,000	16,000	4,000	1,200	18,720	39,920	770
1,200	175,000	40,000	18,700	4,000	1,200	18,720	42,620	820
1,500	200,000	50,000	21,400	5,000	1,200	24,960	52,560	1,010
1,800	225,000	60,000	24,100	6,000	1,200	24,960	56,260	1,080
2,100	250,000	70,000	26,800	7,000	1,200	24,960	59,960	1,145

*Yearly structure cost based on capital recovery factor - 10 percent - 30 years.

**Yearly land cost based on 10 percent interest rate for opportunities foregone.

le at \$11.00 per hour. Using \$3.33 per hour for a driver gives t of \$14.33 per hour of truck use. The average speed is esti- at 16 miles per hour. The other type would be rail-haul from ranafer station to some far distant point for disposal. delphia, Pennsylvania, as reported by PUBLIC WORKS magazine, , 1968), presently does this at a contracted rate of \$5.40 on for shipping and disposal in abandoned coal mines 200 miles n western Pennsylvania. This cost will be used as an te of rail-haul transport and disposal costs.

b) Estimates of tractor trailer vehicle cost per ton.

Define X_F, Y_F = coordinates of the facility

X_D, Y_D = coordinates of the disposal point

c = capacity of the vehicle in pounds

s = travel speed in miles per hour

d = distance to be traveled - one way

l = cost per hour of vehicle plus labor

ost per ton = $(2 \frac{d}{s} \times \frac{l}{c} \times 2000)$

d is measured as before by a rectangular metric

$$d = |X_F - X_D| + |Y_F - Y_D|$$

$|\quad|$ means absolute value of the difference.

c) Costs and locations of disposal areas - The location of sposal points are given as the two existing incinerators, the rd and the Pulaski. One additional alternative, that of rail-

haul, is also included. The unit cost used as based on the
ly mentioned data accounts for a rail-haul of approximately
Therefore, it is not necessary to locate the rail-haul disp
facility. The x-y coordinates for the two existing facilit
the unit cost of operation as given in the 1965 Public Work
of the City of Baltimore are shown in Table 5-8. Refer to
for location in relation to the collection area.

Table 5-8. Characteristics of Baltimore Incinerato

Name	Location		Capacity (tons/day)
Reedbird - #3	x = 26,000	y = 45,000	600
Pulaski - #4	x = 49,000	y = 26,000	800

*From Anon. (1966a), Baltimore Public Works Department Annu

D. Results of the Analysis

1. Explanation of Symbols

A total of 60 [1/] runs were made with the facilities locati model and the data case of section C. These runs represent a b spectrum of changes in parameters and conditions in order to sh the sensitivity of the system and to estimate the order of magn of the cost of proposed system policy and operational changes. runs are summarized in Table 5-9. The headings of each column explained and referenced in the following manner:

Type of Transfer Station - For most runs, seven transfer tation site alternatives were allowed, A, B, C, D, E, F and RR. These symbols and the locations of the sites have been shown reviously in Table 5-6. It was assumed in most runs that the s apacity facility was proposed at each site. Any deviation from his policy, or in the number of sites available, is noted in th EMARKS column beside the run.

Collection Frequency - Two cases were considered: twice eek (2) and three times a week (3).

Collection Costs - These are the costs of picking up the solid waste within the collection areas. The basis for estimati his cost is shown in section C (3) of this chapter. Units are iollars per week.

[1/] Each run took an average of 45 seconds, or for the 60 runs a t of 45 minutes of execution time on the IBM 7094.

Transfer Cost from Collection - This represents t
tation costs incurred by the collection vehicles in haul
collected load from the area where it was picked up to t
where it will be discharged, and the return trip to the
area. The point of discharge in most cases may be eithe
facility or a final disposal point. In most cases when
stations were built not all collection vehicles used the
still economically feasible for some collection subareas
directly to final disposal. Units are dollars per week.

Facility Costs - Cost per week of building and op
facility of a given capacity. Computations to arrive at
figures are shown in Table 5-7.

Transfer from Tranafer Stations - This category r
cost incurred using specialized tractor trailer transpor
to move wastes from the transfer site to the final dispo
rail-haul is used, such transfer costs do not appear in
category but as part of the disposal cost, since there i
break apart the unit cost figure for rail-haul and dispo
are dollars per week.

Disposal Costs - This reflects the unit cost of c
all cases where local disposal was chosen, the Pulaski i
was chosen over the Reedbird incinerator because of its
location with regard to the northwest quadrant that make
study area, and because of the lower unit costs of dispo
newer Pulaski incinerator. In the case where rail-haul

Table 5-9. Description of Runs with Baltimor

Run No.	Type of Trsnsfer Station Proposed Tons/Wk	Collection Freq. Times per Wk	Collection Cost Dollars per Wk	Transfer from Collection Dollars per Wk	Fac'lty Costs Dollars per Wk	Tra f Tra Sta Dol pe
1	1200	2	8,103	1936	820	
2	NONE	3	9,361	4040	0	
3	600	3	9,361	2226	719	
4	900	3	9,361	1847	770	
5	900	2	8,103	2097	770	
6	600	2	8,103	1490	1438	
7	1200	3	9,361	1699	820	
8	1500	3	9,361	1620	1010	
9	1500	3	9,361	1780	1010	
10	NONE	3	9,361	5060	0	
11	NONE	3	9,361	6087	0	
12	1500	3	9,361	1780	1010	[illegible]
13	1500	3	9,361	1780	1010	[illegible]
14	NONE	3	9,361	1773	1010	
15	NONE	3	9,361	1773	1010	
16	1500	3	9,361	1780	1010	[illegible]
17	NONE	2	8,103	4563	0	

Load Sent to Fac'l't's Tons/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Total Cost Dollars/Wk	Remarks
1060	368	1428	15,599	
0	1428	1428	17,399	
600	828	1428	16,844	
900	528	1428	16,606	
900	528	1428	15,598	
600 / 600	228	1428	15,869	
1060	368	1428	16,620	
1093	335	1428	16,754	
1428	0	1428	17,063	Av'ge haul distance 10 miles
0	1428	1428	18,419	Av'ge haul distance 10 miles
0	1428	1428	19,449	Av'ge haul distance 12 miles
1428	0	1428	17,349	Av'ge haul distance 12 miles
1428	0	1428	17,749	Av'ge haul distance 14 miles
1428	0	1428	19,855	Av'ge haul distance 14 miles
1428	0	1428	19,855	Av'ge haul distance 16 miles
1428	0	1428	18,149	Av'ge haul distance 16 miles
0	1428	1428	16,664	

Table 5-9. Description of Runs with Baltimore Data (

Run No.	Type of Transfer Station Proposed Tons/Wk	Collection Freq. Times per Wk	Collection Cost Dollars per Wk	Transfer from Collection Dollara per Wk	Fac'l'ty Costs Dollars per Wk	Trans fro Trans Stati Dolla per
18	1500	2	8,103	1846	1010	76
19	1500	2	8,103	2020	1010	90
20	NONE	2	8,103	5714	0	
21	NONE	2	8,103	1967	1010	
22	1500	2	8,103	2020	1010	13
23	1500	2	8,103	2020	1010	18
24	NONE	2	8,103	1967	1010	
25	NONE	2	8,103	1967	1010	
26	1500	2	8,103	2020	1010	22
27	1800	2	9,245	2028	1080	8
28	1800	2	10,107	2220	1080	9
29	1800	2	10,967	2408	1080	9
30	1800	2	11,795	2590	1080	10
31	1800	2	12,674	2781	1080	11

Load Sent to Fac'l't's Tons/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Total Cost Dollars/Wk	Remarks
1093	335	1428	15,722	
1428	0	1428	16,031	Av'ge haul distance 10 miles
0	1428	1428	17,815	Av'ge haul distance 10 miles
1428	0	1428	18,791	Av'ge haul distance 12 miles
1428	0	1428	16,481	Av'ge haul distance 12 miles
1428	0	1428	16,931	Av'ge haul distance 14 miles
1428	0	1428	18,791	Av'ge haul distance 14 miles
1428	0	1428	18,791	Av'ge haul distance 16 miles
1428	0	1428	17,381	Av'ge haul distance 16 miles
1202	369	1571	17,592	Increase waste load by 10%
1315	402	1717	19,134	Increase waste load by 20%
1427	436	1863	20,670	Increase waste load by 30%
1534	470	2004	22,149	Increase waste load by 40%
1649	504	2153	23,717	Increase waste load by 50%

Table 5-9. Description of Runs with Baltimore Data (

Run No.	Type of Transfer Station Proposed Tons/Wk	Collection Freq. Times per Wk	Collection Cost Dollars per Wk	Transfer from Collection Dollars per Wk	Fac'l'ty Costs Dollars per Wk	Trans fro Trans Stati Dolla per
32	1800	2	13,501	2965	1080	122
33	1800	2	14,356	3212	1080	126
34	1800	2	15,220	3507	1080	126
35	1800	2	16,074	3818	1080	126
36	1800	2	8,103	1700	1080	77
37	1800	2	8,103	1688	1080	45
38	1800	2	8,103	1684	1080	34
39	1800	2	8,915	1846	1080	76
40	1800	2	7,310	1846	1080	76
41	1800	2	6,660	1846	1080	76
42	1500	2	11,264	2918	1010	105
43	2100	2	15,043	3808	1145	141

..'l'ty ..lt	Load Sent to Fac'l't's Tons/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Totsl Cost Dollars/Wk	Remarks
	1756	538	2294	25,198	Increase waste loa[d] by 60%
	1800	639	2439	26,737	Increase waste loa[d] by 70%
	1800	706	2586	27,869	Increase waste loa[d] by 80%
	1800	931	2731	29,818	Increase waste load by 90%
	1112	316	1428	15,659	Transfer vehicle sp[eed] 20 mph
	1135	293	1428	15,323	Transfer vehicle sp[eed] 30 mph
	1147	281	1428	15,209	Transfer vehicle sp[eed] 40 mph
	1093	335	1428	16,604	Vary collection rat[e] by -10%
	1093	335	1428	14,999	Vary collection rat[e] by +10%
	1093	335	1428	14,349	Vary collection rat[e] by +20%
	1500	653	2153	22,270	B is only alternati[ve] Increase waste lo[ad] by 50%
	2100	775	2875	24,516	B is only alternati[ve] Increase waste lo[ad] by 100%

Table 5-9. Description of Runs with Baltimore Data

Run No.	Type of Transfer Station Proposed Tons/Wk	Collection Freq. Times per Wk	Collection Cost Dollars per Wk	Transfer from Collection Dollars per Wk	Fac'l'ty Costs Dollars per Wk	Trans fr Trans Stat Doll per
44	1500	3	9,361	1620	1010	7
45	1500	2	8,103	1498	2020	8
46	1500	3	9,361	1230	2020	5
47	1500	2	8,103	1754	1010	9
48	1500	3	9,361	1780	1010	7
49	1500	2	8,103	1846	1265	7
50	1500	3	9,361	1620	1265	7
51	1500	2	8,103	1846	1518	7
52	1500	3	9,361	1620	1518	7
53	1500	2	8,103	4564	0	
54	1500	3	9,361	4040	0	
55	1500	3	9,828	1700	1010	8
56	1500	3	10,291	1780	1010	8

y	Load Sent to Fsd'l't's Tons/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Total Cost Dollars/Wk	Remarks
	1093	335	1428	16,752	B is only alternative Increase waste load 0%
	$\frac{746}{411}$	271	1428	16,471	Political jurisdiction case
	$\frac{700}{415}$	313	1428	17,200	Political jurisdiction case
	1044	384	1428	15,805	Deny B as an alternative
	1428	0	1428	16,863	Deny B as an alternative
	1093	335	1428	15,977	Increase fixed costs by 25%
	1093	335	1428	17,009	Increase fixed costs by 25%
	1093	335	1428	16,230	Increase fixed costs by 50%
	1093	335	1428	17,262	Increase fixed costs by 50%
	0	1428	1428	16,664	Increase fixed costs by 100%
	0	1428	1428	17,399	Increase fixed costs by 100%
	1148	351	1499	17,539	Increase waste load by 5%
	1202	369	1571	18,321	Increase waste load by 10%

Table 5-9. Description of Runs with Baltimore Data (

Run No.	Type of Transfer Station Propoaed Tons/Wk	Collec-tion Freq. Times per Wk	Collec-tion Cost Dollars per Wk	Transfer from Collec-tion Dollars per Wk	Fac'l'ty Costs Dollars per Wk	Trans fro Trans Stati Dolla per
57	1500	3	10,758	1861	1010	88
58	NONE	3	9,828	4245	---	--
59	NONE	3	10,290	4443	---	--
60	NONE	3	10,757	4646	---	--

ad t to l't's s/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Total Cost Dollars/Wk	Remarks
57	384	1641	19,103	Increase waste load by 15%
0	1499	1499	18,270	Increase waste load by 5%
0	1571	1571	19,132	Increase waste load by 10%
0	1641	1641	19,998	Increase waste load by 15%

as the disposal method, the disposal costs include the
hauling to the distant disposal point. Where local dis
chosen, the cost includes only the actual disposal proc
sourcea of these costs are ahown in section C (6) of th
and are in dollars per week.

<u>Facilities Built</u> - If transfer sites were chosen
models, this indicates at which sites transfer stations
given capacity would be built.

<u>Load Sent to Facilities</u> - Tons of waste per week
collection areaa that would be sent to transfer facilit
than directly to final disposal.

<u>Load Directly to Disposal</u> - Tons per week sent d
disposal without trans-shipment to a transfer site.

<u>Total Load</u> - The total waste load in tons per we
Represents the aum of the previous two categories.

<u>Total Cost</u> - The sum of the collection, transfer
collection, facility, transfer from facilities and diap
Units are dollars per week.

2. Explanation of Runs

<u>Estimation of Present System Costs and Verificat</u>
important aspect of the study is to establish some benc
which the results of changing the system may be measure
their efficiency. The present system runs with two tim
collection frequency so the computation of this cost wi

bench mark but also as a check on the validity of the nce published figures are available for the cost of running l system (Anon, 1966a). Run number 17 represents this ion since its data inputs are the best estimates of all lection frequency is twice a week and no transfer stations ed. The yearly cost of collection by the city from the division for the year 1965 was $722,000. The model nis cost as $12,666 per week, or $688,632 per year, which erence of 4.5 percent. The difference between the two is slight and may be explained by the fact that 1960 n estimates were used in the model while the actual coat presents the cost of collecting from the 1965 population ne region. Since the northwest division of the city, ly at its outermost fringes, is gaining in population, it xpected that the model estimates would be slightly lower.

imation of a Three Timea a Week Collection Frequency Bench

run number 2, an estimate was made of the cost of three ek collection frequency without transfer stations, with a total system cost of $17,399 per week. When compared to a week bench mark of $16,664 per week this showa only a t increase in cost to increase in collection frequency. his estimate is really only a lower bound on the actual ree times a week collection and thus should be viewed with re are several assumptions buried in the calculation of

the cost of three times a week collection that should be stated. The first is that the routes for the vehicles wi changed from those of the twice a week collection vehicle insure that the truck is close to capacity before it make to the transfer or disposal point. As noted, there is no mentation on a vehicle which tells the driver the present his vehicle. Routes are usually assigned in terms of def with the driver returning when he reaches a certain place in three times a week collection, less waste is generated area as there has been less time since the last collectio vehicle must travel farther to get a complete load. The vehicle is an important factor in the calculation. The a load for twice a week collection was 9000 pounds. Should average load for three times a week collection drop becau are improperly designed, say by 10 percent, the cost dif between twice and three times a week would rise to 8 perc the average collected weight drop by 25 percent, the cost between twice and three times a week collection would ris cent. Thus extreme care must be taken in deaigning the r make sure that enough load is picked up before transporta factor to be watched in the design of the routes is the t Moat operations work under a "no overtime" constraint, wh that at a certain time the truck will return, regardless present load. Since it takes longer to collect under a t a week policy careful attention must be paid to the time

s. Models as suggested in Chapter IV on routing would be ssign routes correctly.

second assumption that has been made in these computations he waate load is the same for twice a week and three times llection. This may not be a valid assumption, but no evidence exists to disprove it. However, work by Quon et) based on actual observations in a controlled experiment o shows significant increases in waste loads when collec- uency was increased from once to twice a week. They found d increases in the order of 30 to 50 percent. To test the waste load increase on the three times a week collection on, runs 58, 59 and 60 were made. These runs represent es a week collection with no transfer, and a waste load of 5, 10 and 15 percent respectively. These runs show that tion of the difference between twice and three times a ction is quite sensitive to the assumption made about . These computations are shown in Table 5-10. Great care taken in estimating the difference between twice and three ek collection, as the cost difference is very sensitive waste load estimates.

ever, in measuring and comparing differences between indi- s when both have the same collection frequency, the type tion made does not have a serious effect on the calculation same assumptions appear in both. The remainder of the runs a collection frequency of three times a week are all made

with the assumption that the average truck load does not c
from twice a week collection and that individual waste loa
generated does not change.

Table 5-10. Different Waste Load Assumptions and Their Ef
the Percent Difference Between Twice and Thre
a Week Collection with no Transfer

Waste Load Assumption	Total System Cost Dollars per Week	Cost Diffe Between Twic and Three T Week Colle
Same as 2 times a week	17,399	4.5%
5% greater	18,270	10.0%
10% greater	19,132	15.0%
15% greater	19,998	20.0%

<u>Is Transfer Feasible?</u> - With the best data availab
various cost parameters and for the cost of facilitiss, a
runa was carried out with differsnt capacity transfer stat
the proposed transfer sites to show if tranafer stations l
cost of collection. In every case they did. The results
analysis also give some interesting insights into the syst
3, 4, 7 and 8 were made for three times a week collection
different sized transfer stations ranging from 40 percent

weekly waste load (600 tons per week) to over 100 per-
ons per week). Similarly, runs 6, 5, 1 and 18 were
twice a week collection rate and the results of all
re presented in Figure 5-4. First, notice that beyond
per week alternative, the cost function for the higher
tions is more favorable and quite flat, indicating little
to size within that range. More important, for each
y one transfer site was chosen and for all six cases
s B. Thus this preliminary analysis would seem to favor
esent system. In all cases, the building of only one
tion indicated that some waste load was still going
the final disposal point without transfer. Both the
0 ton capacity per week facilities were used at less
y with about 40 percent of the waste in both caaes going
disposal. Later runs will show that this proportion is
ve to the location of the final dispoaal points. The
ee times a week collection with transfer is compsrable
sent two times per week collection without transfer.
ations will show how comparable these costs are if
hould be increased for the higher frequency case.
e smaller capacity (600 tona per week) stations, a
ernative (A) was choaen and in one caae two alternatives
e chosen. In general, the difference between the solu-
ansfer stations and the one without is in the range of

4 to 7 percent with the two times a week collection freq allowing a better decrease. The absolute difference bet transfer and non-transfer solution is a rough measure of facility costs can increasc before transfer is no longer More will be said sbout this later.

Effect of Increased Haul Distance to Final Dispos present average haul distance from the northwestern divi disposal at the Pulaski incinerator was shown to be 8 mi by Truitt (1968) and was verified by this author. Now s the haul distsnce was increased. Such a case is possibl the city expands, the tendency is to move waste disposal in order to avoid creating a nuisance to nearby neighbor because of the lack of availability of unused sites near Runs were made to see what the effect would be if the di points were moved so that average haul distance was incr 12, 14 and 16 miles. In this analysis there is an addit possibility which was not considered in Truitt's work. haul to far distant disposal points. As average haul di increases, transfer facilities become more and more favo finally the most dominant choice is rail-haul. A set of made to see how moving the disposal point would affect a and without tranafer atations. Runs 10, 11, 14 and 15 r case of increasing haul distance without allowing transf three times a week collection frequency. Runs 9, 12, 13

sfer under the same conditions. Figure 5-5 shows the f these runs compared to the cost of the rsil-haul alter- It is evident that as the haul distance increases, becomes more favorable to non-transfer. In fact with non- rail-haul becomes feasible at 12.9 miles average haul If transfer is allowed, rail-haul is not s suitable e until almost 26 miles average haul distsnce. This that as pressure mounts to move disposal out farther from a transfer facility system would preclude rail-haul for given that the relative costs of the alternatives would same. Further, the transfer site chosen in each of the he same site C, indicating that the site selection in is quite stable. Also, as soon as the disposal point has only 2 miles, all waste in the system goes through the acilities and none is taken directly to the disposal point. e reason C was chosen for this case while B was chosen for us cases with present location of facilities. The choice and C favors B only slightly, and when conditions are cause more load to go through a facility, C becomes the native.

Figure 5-6, results of runs 19, 20, 21, 22, 23, 24, 25 and tted to show how a twice a week collection system would r the same haul distance changes. The results are quite ith C being chosen as the transfer site and the rail-haul e becoming feasible at a slightly smaller haul distance

than in the previous case.

Thus it would appear that the questions concerning transf and rail-haul are intimately linked. Under present conditions, rail-haul is not a feasible alternative. At a cost of $18,791 p week for the present system it represents a 12.8 percent increas cost over the system with no transfer, and an 18.3 percent incre over the system with transfer. However, if haul distance should increase, rail-hsul would become a feasible alternative in a fa short time if no transfer is established. If transfer faciliti are built they reduce costs to the extent that rail-haul would n be feasible for a considerable time. As for transfer itself, it does exhibit some, but not extraordinary, savings over non-trans for the present system. For future conditions, however, these savings will increase, thus making transfer even more favorable.

<u>Sensitivity to other Parameters</u> - Other runs were made to show how sensitive the system was to estimates of tha various parameters. Since waste load estimation in three times a week collection has been shown to have an effect on the solution, sev additional runs were made to study this situation. Runs 55, 56 57 represent runs with transfer facilities allowed, three times week collection frequency and an increase in the waste load of 5 and 15 percent, while runs 58, 59 and 60 represent tha system without transfer. The results are shown in Tabla 5-11. The cos of the transfer station decision is even more sensitive than the

e 5-11. Different Waste Load Assumptions and Their Effect on Percent Difference Between Twice and Three Times a Week Collection with Transfer

ease in e Losd %)	Increase in Cost with no Transfer (%)	Increase in Cost with Transfer (%)	Site Alternative Selected
0	4.5	6.4	B
5	10.0	11.4	B
10	15.0	16.6	B
15	20.0	21.5	B

transfer solution. However, in all cases the site selected was ndicating that only the cost estimate but not the site chosen ensitive to this parameter. Twice a week collection was tested its sensitivity to waste load increase in runs 27, 28, 29, 30, 32, 33, 34 and 35, where the waste load was progressively eased until it almost doubled. Results of these runs are shown igure 5-7. Even though waste load was increasing, the solution responded linearly and the same alternative B, was chosen for solutions. Even when waste load was almost double the transfer ion capacity it was not optimal to build a second facility at fferent site.

Runs 36, 37 and 38 show how sensitive the solution is to the

apeed of the transfer vehicle from the transfer facility to t disposal point. Resulta are plotted in Figure 5-8. Present mates place this speed at 16 miles per hour since this ia the approximate speed in traffic of the collection vehiclea, and transfer vehicles are larger tractor trailers which must oper under largely the same traffic conditiona. The plot indicate an increase in the traffic speed from 16 to 30 miles per hour bring about a decrease of about 7.5 percent in the coat of th solution. However, it would be expected that the cost of the remedial meaaures necessary to bring about this increase in s would more than outweigh any savings in transportation costa.

An attempt was also made to see how collection rates a the solution. In runs 39, 40 and 41 collection rates are var by -10 percent, +10 percent and +20 percent. Increasing or d the collection rate by 10 percent leada to about a 5 percent in the solution cost, which falls into the range of being son aensitive. Thus care should be taken in estimating theae rat Further, work rules that tend to increase collection rate wit too great an increase in cost or inexpensive time aaving devi would also bear some investigation.

The effect of changing the estimates of the facilities ia also of interest. This gives some feeling for how careful estimates have to be made for preliminary studies. Runa 49, 52, 53 and 54 show how fixed cost increasea of 25, 50 and 100 would affect the aolution. At about a 75 percent increase in

costs, transfer facilities are no longer feasible, but up to that point the same facility was chosen each time. A rough estimate of how much facilities costs could increase could have been found fro earlier runs simply by looking at the absolute difference in cost between the transfer and non-transfer solutions. If the facility cost increase exceeded this difference, transfer was no longer feasible.

<u>Political, Aesthetic and Regional Constraints</u> - Many times constraints occur which preclude the use of an alternative that would otherwise seem to be the best from an economic point of view Examples are political constraints when a site might better be put to use for other municipal functions, aesthetic constraints where the neighborhood around a site strongly opposes a facility there, and regional constraints which do not allow for cooperation betwee different political subdivisions. Consider first an example where a particular site is not allowed to be used. The results of the analysis to this point show that B is the best transfer site. Run 47 and 48 show what happens when this site ia excluded from the se of alternatives. In the case of two times a week collection, Site is chosen and the increase in the cost is about $60 per week. For three times a week collection the site chosen is site C and the difference is about $110 per week. Both of these changes are minuscule compared to the total cost of operating the system. Thi indicates that there may be readily available alternatives to the

best site thst will not raise the cost of the solution signifi Thus the cost of satisfying additional constraints which prohi certain sites is not particularly high, and the decision maker a great deal of leeway in choosing between alternatives when c criteria must also be considered.

The effect of regionalization will only be treated briefl this example, as neither the time nor data were available for assembling an extremely large regional case. However, the que can be looked at implicitly by assuming that the northwestern division is a unified region, and asking how much costs would increase if it were subdivided into two non-cooperating region Suppose the region were divided into two so that the northern could use only transfer sites A, D and E, and the southern are could use only sites B, C and F. The results of these runs ar shown in Table 5-12. In this case subdividing the region stil makes transfer feasible, but only at an extremely slight advan over no transfer at all. This would suggest that regionalizat would mean increased savings. Although this analysis was not extended to the entire city, it might be expected that each of administrative divisions would not have a transfer facility, b only two or three would be built.

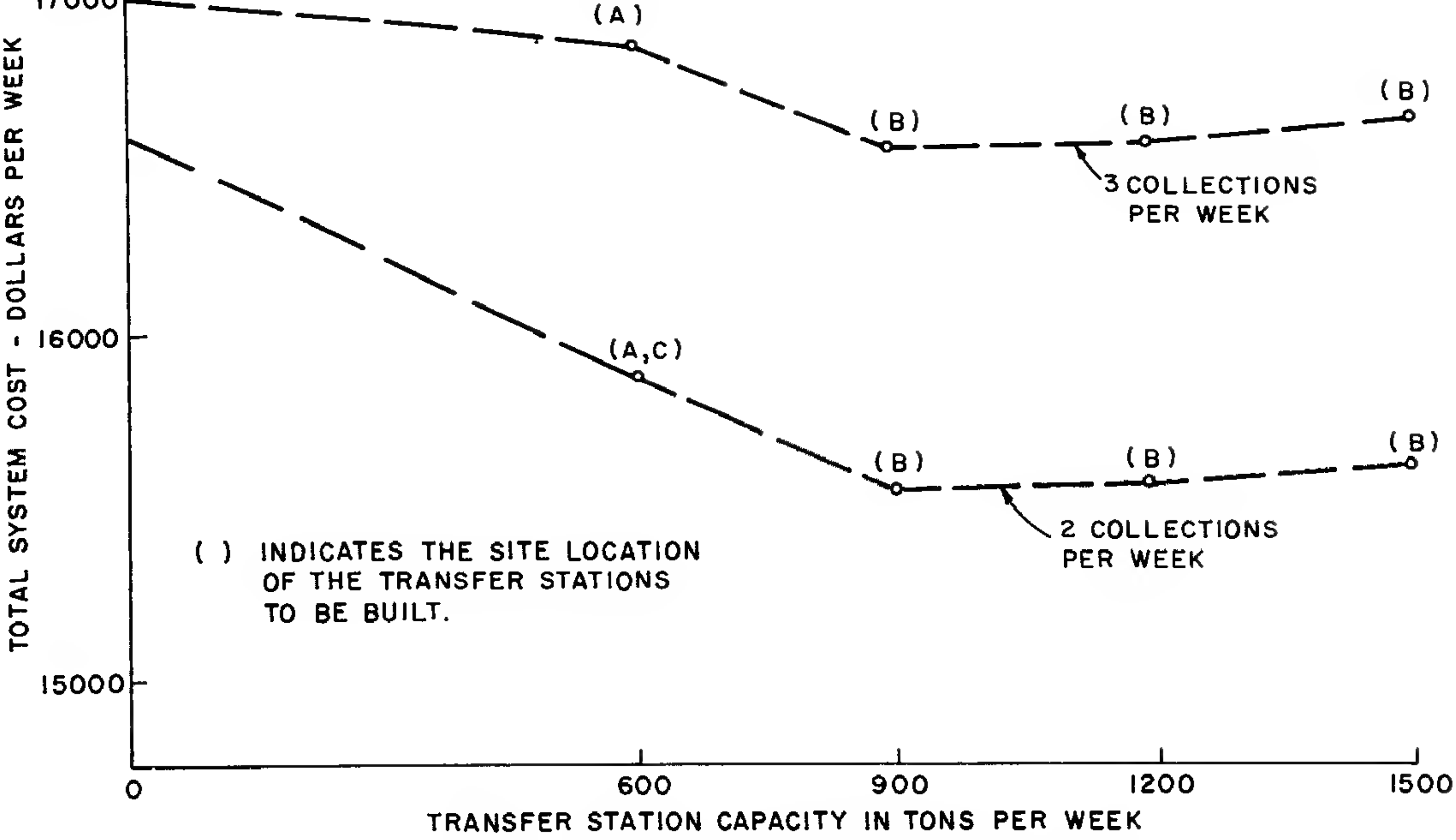
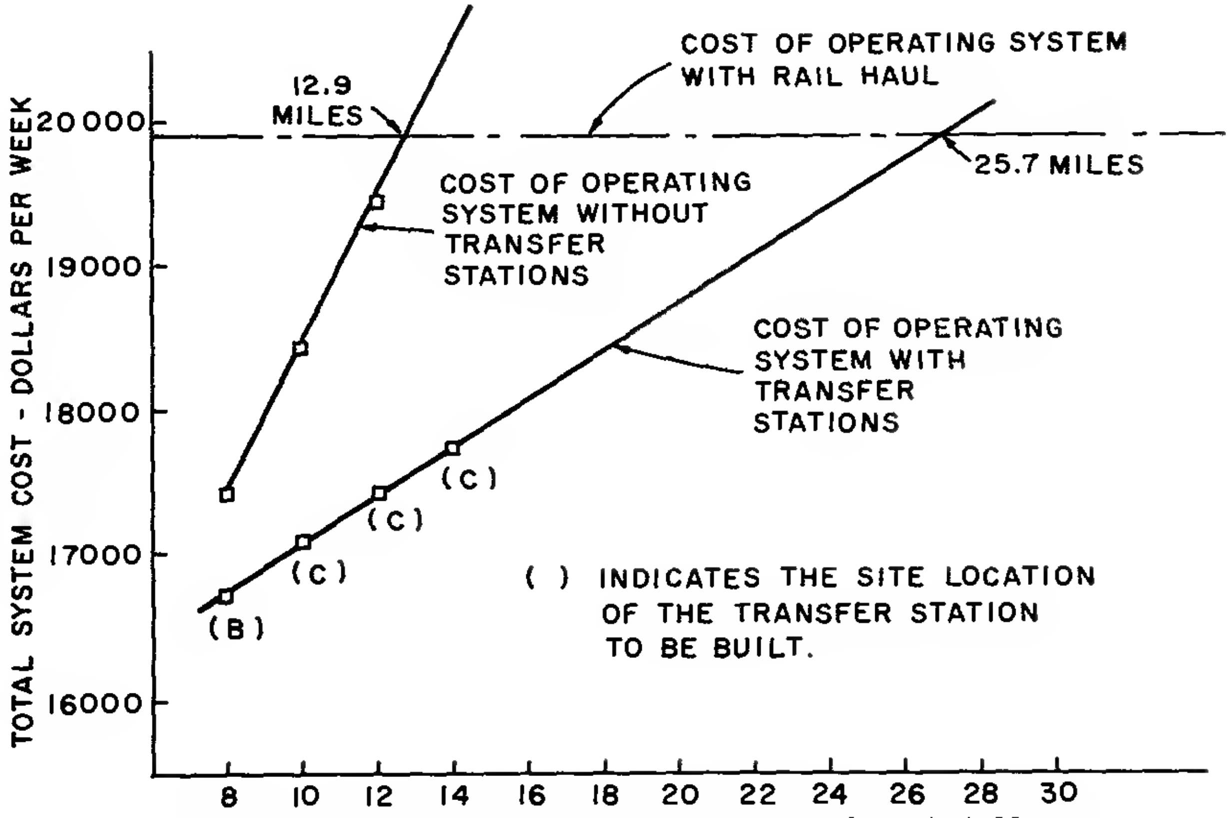


Figure 5-4. Plot of Total System Cost vs. Size of Transfer Station


TOTAL SYSTEM COST - DOLLARS PER WEEK
20000
19000
18000
17000
16000
8
10
12
14
16
18
20
22
24
26
28
30
12.9 MILES
25.7 MILES
COST OF OPERATING SYSTEM WITH RAIL HAUL
COST OF OPERATING SYSTEM WITHOUT TRANSFER STATIONS
COST OF OPERATING SYSTEM WITH TRANSFER STATIONS
(B)
(C)
(C)
(C)
( ) INDICATES THE SITE LOCATION OF THE TRANSFER STATION TO BE BUILT.

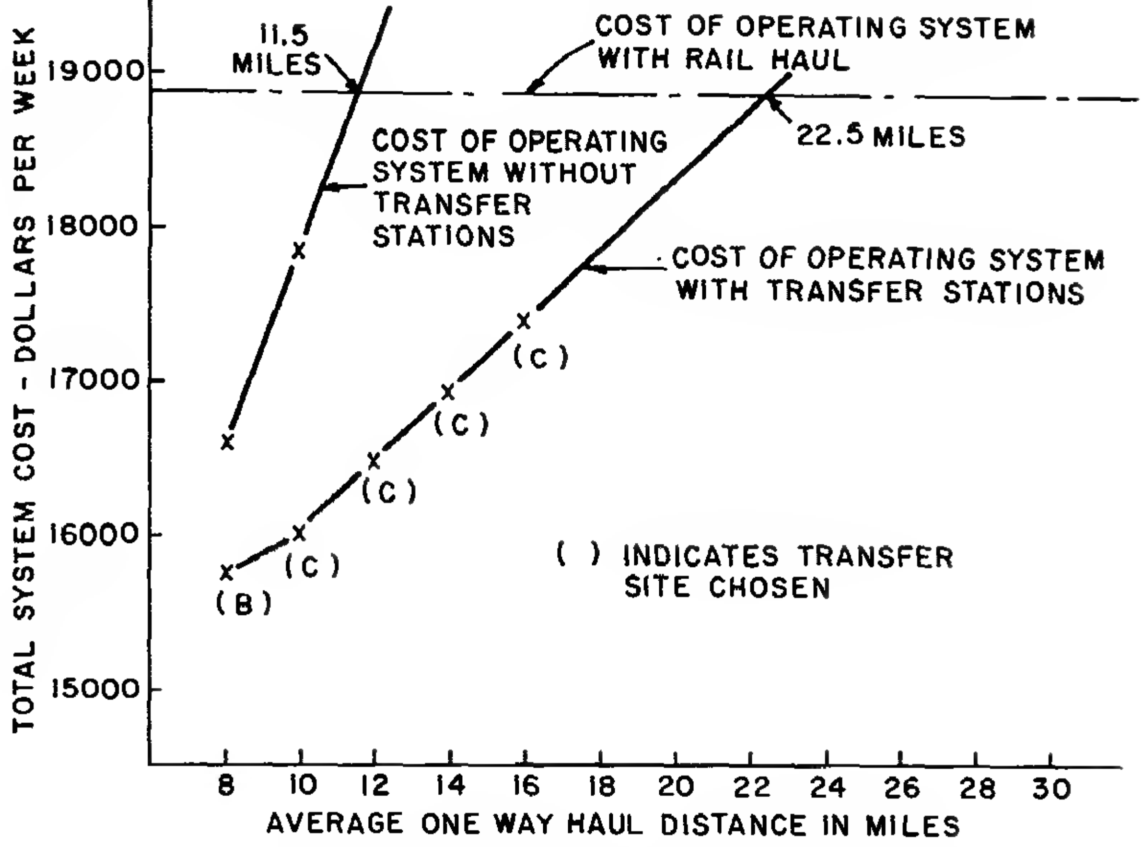
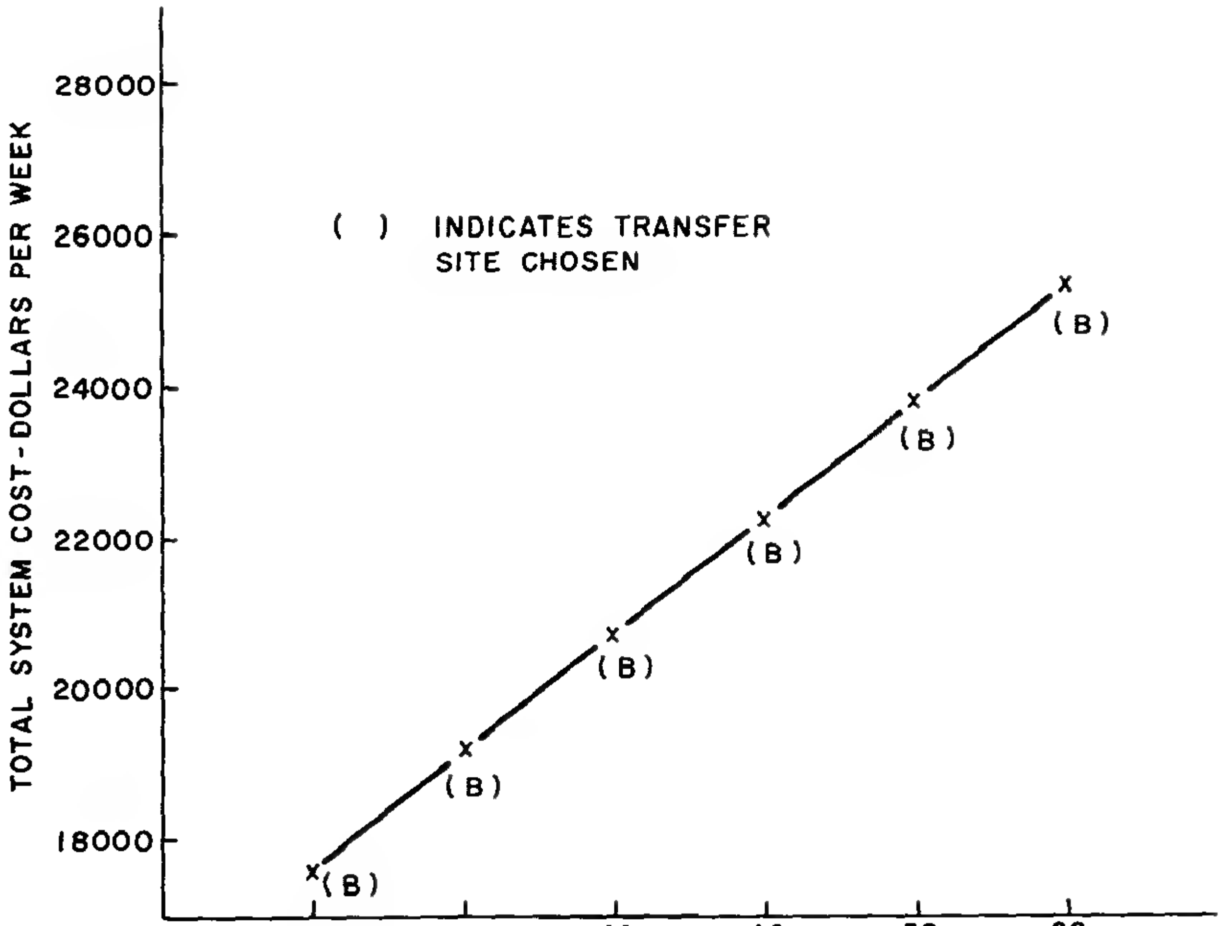


Figure 5-6. Plot of Average Haul Distance vs. Cost of Total System. Collection Frequency = Two Times per Week.



TOTAL SYSTEM COST - DOLLARS PER WEEK
28000
26000
24000
22000
20000
18000
( ) INDICATES TRANSFER SITE CHOSEN
(B)

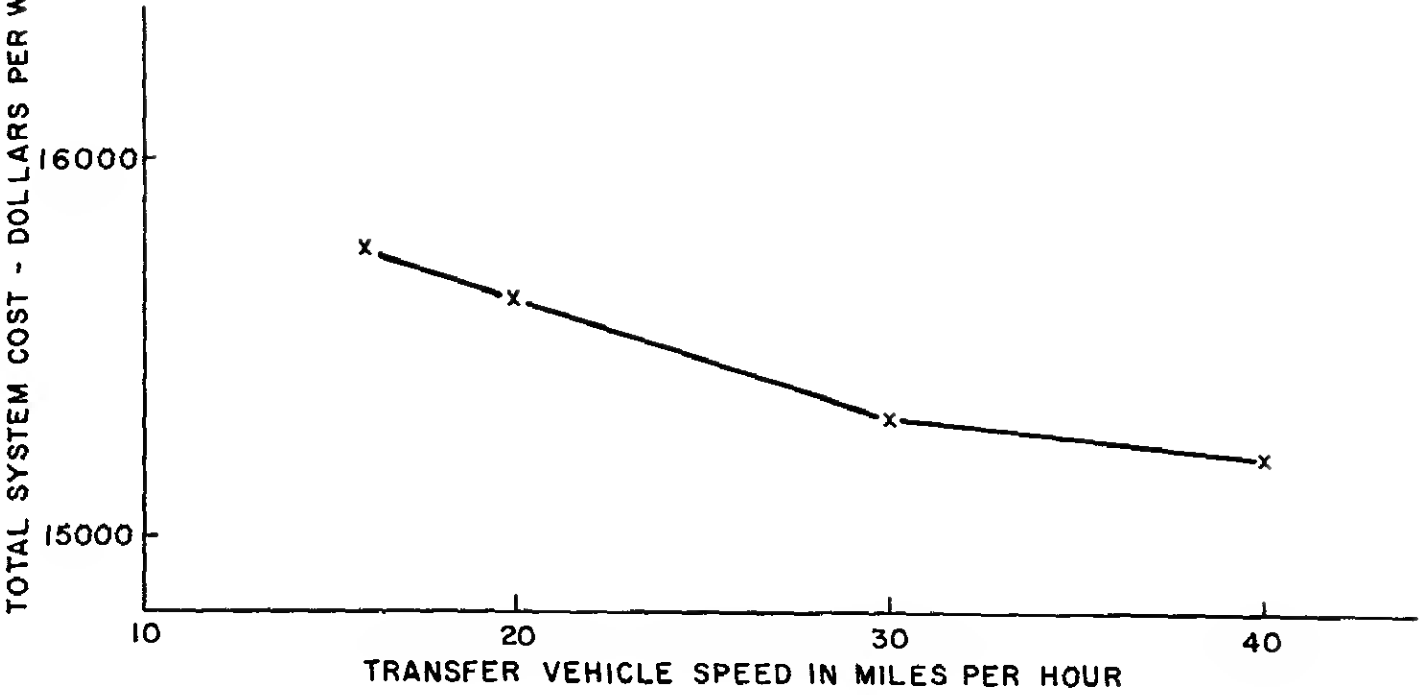


Figure 5-8. Plot of Transfer Speed in MPH vs. Cost of Solution

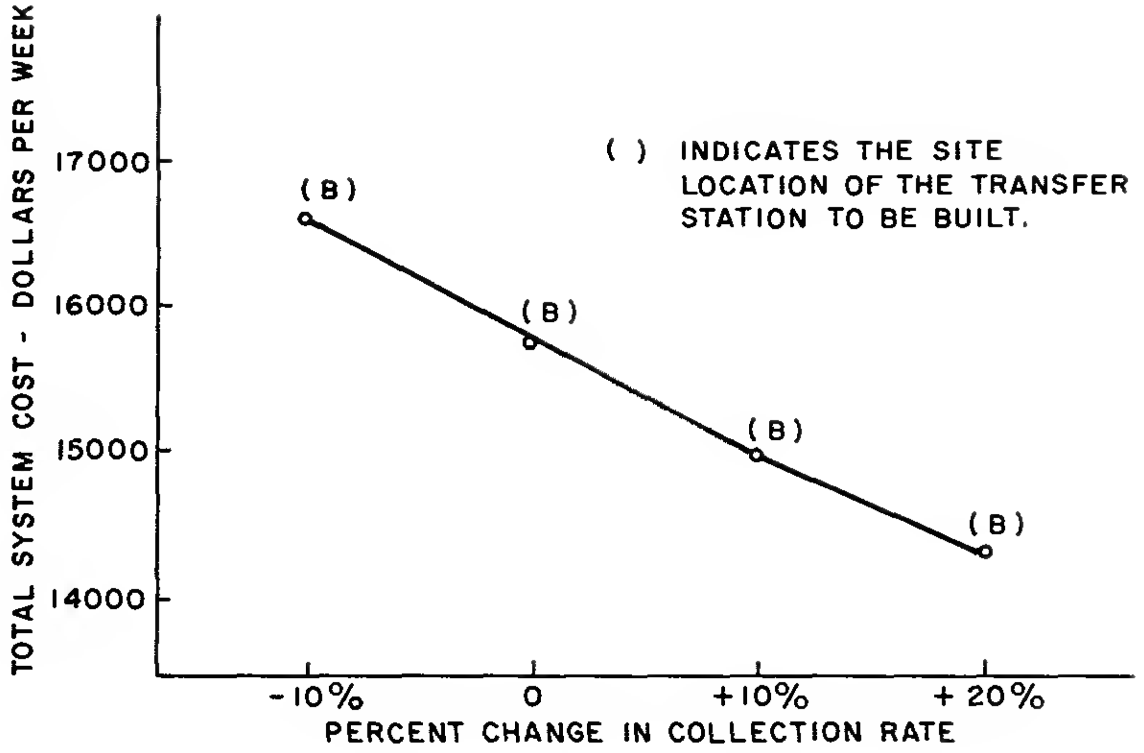


Figure 5-9. Plot of Varying Collection Rate vs. Cost of Solution.

Table 5-12. Effects of Regional Cooperation

Collection Frequency	Northwestern Division is Two Districts with Transfer		Northwestern Division is One District Without Transfer		Northwestern Division Without Transfer
	Cost/Week	Sites	Cost/Week	Site	Cost/Week
2	$16,471	B and D	$15,720	B	$16,664
3	17,200	C and F	16,752	B	17,399

Summary of the Experimental Analysis

The following general statements about the system can be e, based on the rough analysis with estimated data for the thwestern division. These runs have been intended as a demonation of the techniques of this thesis, rather than a detailed lysis of the system. The actual findings would need verifica- n with better cost data before they could be viewed as pletely reliable.

1. The building of transfer facilities appears to be feasible and would result in an annual savings, under present conditions, of 7 percent in the total cost of operating the solid waste collection system. Thia saving would be expected to increase in the future, as the system expands and grows.

2. For the northwestern division only one tranafer facility would be built, with site B chosen aa the location. Site C is also a good alternative and selecting C over B would show little disadvantage.

3. The most favorable size of the facility to be built is 1500 tons per week capacity. At this size aome of the waste material will be taken to transfer and aome will still be taken directly to the disposal point. Theae assignments are shown by the model.

. A rail-haul alternative would cost about 12 percent more to operate than the present system and about 18 percent more than the system with a transfer facility. Haul distance would have to increase an average of from 3.5 to 5 miles for rail-haul to be favorable without transfer, and from 14 to 18 miles to be favorable with transfer.

. The cost of changing from two times a week frequency of collection to three times a week is difficult to estimate because of the sensitivity of this calculation to estimates of how much the waste load would be with three times a week collection. Estimates of the added cost range from 4.5 percent if there is no increase in the waste load, up to 21.5 percent with a 15 percent increase in the waste load. All of these figures are based on the assumptions that routes for vehicles under three times a week collection be redesigned for efficiency purposes. It is strongly advised that more studies of the waste load question in this region be carried out before any firm decision be made on changing collection frequency.

. While differences in assumptions make estimating the cost of some programs difficult, the site selection B, for a transfe facility is remarkably insensitive to parameter change. Regardless of the collection frequency, site B is the best sit for the transfer facility.

7. The system shows some sensitivity to collection rates, which indicates that care should be taken in measuring them and investigation of means of altering them will be of some help.

8. If for some reason site B could not be used, site C first and then site A are good alternatives, and it would cost little extra to change the selection to them. The effects of regionalization are such that the northwestern division is better operated as one diviaion than as two. It is further expected that extension of this type of analysis to the entire city might show even greater efficiency with transfer, and this is recommended as the next step in the analysis procedure.

9. The amount of computer time necesaary for this analysis of 60 runs was 45 minutes, using the IBM 7094. At commercial rates of $500 per hour, this represents a cost of approximately $375.

CHAPTER VI. CONCLUSIONS AND EXTENSIONS

The goal of this thesis has been the development of too gain a better understanding of a large-scale public system. T it is on the basis of the results of the example rough analysi the Baltimore, Maryland, solid waste collection system in Chap that the success of such an undertaking should be measured. W most of the data used for the analysis were less than elegantl obtained, a firm body of facts and conclusions emerge from the exercise. Answers to many of the basic questions with which t analyst is concerned were found. Most important, a feeling fo sensitivity of the system to many of its parameters was estsbl The decision to change from twice a week to three times a week collection was shown to be quite sensitive to estimates of was generation under the two regimes. Thus, it is recommended tha additional study in this area be carried out before a decision made about a policy change. On the whole, however, the proble choosing transfer facility location appears to be quite stable the same location being elected under a severe change in syste conditions.

Some drawbacks in the analysis were encountered because the need to subdivide the problem in order to solve it. At on point it was necessary in the study of facility location to as that the new routes for vehicles would be set up efficiently i service policy whould change. However, the model could not

determine how this was to be done, and the problem would have to looked at as a separate vehicle acheduling problem as described Chapter IV.

From a computational point of view the analyais in Chapte was a success. Sixty different problems, each involving generat data for forty waste sources and considering nine intermediate alternatives, were solved in about three-quarters of an hour on IBM 7094. This represents a cost at commercial rates of leas th 400 dollars, or about two weeks' salary for a medium level engineering analyst. In terms of the information generated, it seems like a considerable bargain.

Looking at the individual models developed, the large-aca "flow of goods" models developed for the facility location probl of Chapter II and the multi-commodity truck assignment problem o Chapter III proved to be the most successful. When programmed f the computer, they were of sufficient size and speed to handle a very large complex system and find solutions quickly. Since muc of the value of the models is in repeated aolution with changing parameters to determine sensitivity, efficient solution time is great importance. The vehicle scheduling models of Chapter IV d not display the same qualities. People working in this field ha uniformly found that securing an optimal aolution to even a smal scale problem in a reasonable amount of time was difficult to impossible. Although the author's work in a problem area of thi general category extended theoretical concepts involving the use

n one terminal, the ssme type of problem occurred. Thus, e no existing optimization techniques that will allow the ation of a city-sized routing problem. The development stic methods has advanced, however, and feasible solutions e complex problems can be found quickly. No one has yet ated the difference between a good feasible solution and al solution in vehicle routing to determine if the search mal procedures is worthwhile. Meanwhile, sensitivity using heuristic rather than optimal procedures must be ith extreme caution and care.

erhaps the problems of routing would benefit from a shift sis from discrete problems to continuous problems. Vehicle ng is viewed as a problem of finding routes between points or in terms of a network example, in finding which travel to visit all nodes. The rapidly incressing orial nature of the problem is what makes computation of solutions so difficult. Studying public services in city etworks would reveal a continuous diatribution of demand arc with a requirement that all arcs be traveled. For cases, this turns out to be a problem that is remarkably quick to solve. The chinese postman problem asks how a s route for one vehicle can be found through a network vels all arcs and minimizes total distance traveled. The of these techniques to problems of more than one service

vehicle, each with a capacity constraint, while retaining the property of efficient solution would mean a dramatic increase in the type of analysis done on vehicle routing in solid waste collection.

The overall conclusion of this thesis is that models such as developed, limited though they may be by applying only to sub-problems of the total collection system, can still provide a great deal of insight into the system. The types of questions amenable to study using these techniques, provide a wealth of information to the decision maker charged with the operation of the system. It is hoped that such models will become increasingl valuable tools in the management and planning of large-scale soli waste collection systems.

BIBLIOGRAPHY

on, L. E., A mathematical model for the optimization of a
es management system, Sanitary Engineering Research Laboratory,
ersity of California, Berkeley, SERL Report 68-1, February 1968.

G. M., and J. R. Hamann, Truck routing: a branch and bound
tion, mimeo, presented at the meeting of the Operations
rch Society of America, September 1967.

us, California waste management study: report to the State
lifornia, Department of Public Health, by Aerojet General
ration, Azuza, California, 1965.

us, The annual report of the Department of Public Works for
ear ending December 31, 1965, Department of Public Works,
more, Maryland, 1966.

us, The applicability of a systems approach to solid waste
ement problems, report presented to the Maryland State
tment of Health, Division of Solid Wastes, by Management
ology, Inc., July 1966.

us, System 360 vehicle scheduling program (360A-ST-06X)
cation description, International Business Machines Corp.,
ical Publications Department, White Plains, New York, 1968.

us, System 360 vehicle scheduling program - program description
perations manual (360A-ST-06X), International Business Machines
, Technical Publications Department, White Plains, New York,

us, Refuse disposal via railroads, Public Works, July 1968.

E., An additive algorithm for solving linear programs with
one variables, Operations Research, 13, 517-546, July-August

E., Discrete programming by the filter method, Operations
rch, 15, September-October 1967.

, M. L., On finding integer solutions to linear programs,
matica, Princeton, N. J., 1964.

, M. L. Integer programming: methods, uses, computation,
ement Science, 12, November 1965.

Balinski, M. L., and R. E. Quandt, On an integer program for a delivery problem, Operations Research, 12, 300-304, 1964.

Barton, R. A., and R. J. Gumaer, The optimum routing for an air cargo carrier's mixed fleet, presented at the Sesquicentennial Forum on Transportation Engineering, New York, N. Y., August 28 30, 1967. American Society of Mechanical Engineers, publicatio 67-TRAN-18.

Baumol, W. J., and P. Wolfe, A warehouse location problem, Operat Research, 6, 1958.

Bellmore, M., List processing subroutines, mimeo, The Johns Hopki University, February 1968.

Bellmore, M., and G. Nemhauser, The traveling salesman problem - survey, Operations Research, 16, May-June 1968.

Benders, J. F., Partitioning procedures for solving mixed variabl programming problems, Numerische Mathematik, 4, 238-252, 1962.

Berge, Claude, The theory of graphs and its applications, John Wi and Sons, Inc., New York, 1966.

Black, R. J., The solid waste problem in metropolitan areas, California Vector Views, 11, September 1964.

Bomberault, A. M., and W. W. White, Networks and transportation: the empty freight car allocation problem, IBM New York Scienti Center, Technical Report No. 320-2952, July 1968.

Clarke, G., and J. W. Wright, Scheduling of vehicles from a centr depot to a number of delivery points, Operations Research Soci of America, 12, 568-581, 1964.

Clasen, R. J., The numerical solution of network problems using out-of-kilter algorithm, The Rand Corp., RM 5456-PR, Santa Mon California, March 1968.

Coyle, R. G., Ph.D. thesis, Bradford University, England, 1967.

Coyle, R. G., and M. J. C. Martin, Case study: the cost minimiza of refuse collection operation, ORSA Meeting, May 1968.

Dantzig, G. B., and J. H. Ramser, The truck dispatching problem, Management Science, 6, 81-91, October 1959.

Durbin, E. F., and D. M. Kroenke, The out-of-kilter algorithm: a primer, The Rand Corporation, RM 5472-PR, Santa Monica, Califo December 1967.

, J., Maximum matching and a polyhedron with 0, 1 vertices, nal of Research of the National Bureau of Standarda, B. ematics and Mathematical Phyaics, 69B, January-June 1965.

, J., and E. Johnson, Private communication with ellmore, 1969.

on, M. A., and T. L. Ray, A branch and bound algorithm for location, Operations Research 14, 361, 1966.

, E., F. A. Lehrer, and T. L. Ray, Warehouse location under nuous economies of scale, Management Science, 12, 670-684, 966.

. R., Jr., Constructing maximal dynamic flows from static , Operations Research, 6, 419-433, 1958.

. R., Jr., and D. Fulkerson, Flow in networks, Princeton rsity Preas, Princeton, N. J., 1962.

on, D. R., An out-of-kilter method for minimal cost flow ems, Journal of the Society for Industrial and Applied matics, 9, 18-27, 1961.

el, R. S., and G. Nemhsuaer, Set covering with equality raints, Paper IIG.4, 35th National Meeting, Operations rch Society of America, Denver, Colorado, July 1969.

W. W., H. W. Crandall, J. B. John, and R. A. Spellman, cation of linear programming in the oil industry, ement Science, 3, 407-430, 1957.

on, A. M., Integer programming by implicit enumeration and ' method, The Rand Corporation, RM-4793-PR, Santa Monica, ornia, February 1966. Also published in SIAM Review, 9 30, April 1968.

on, A. M. Implicit enumeration using an imbedded linear am, The Rand Corporation, RM-5406-PR, Santa Monica, ornia, September 1967.

F., A multiphase-dual algorithm for the zero-one integer amming problem, Operations Research, 13, 879-919, November-ber 1965.

F., Finding an optimal edge-covering tour of a connected (the Chineae postman's problem), Operations Research Center, rsity of California, Berkeley, ORC 67-13, March 1967.

Golueke, C. G., and P. H. McGauhey, Comprehensive studies of sol wastes management, first annual report, Sanitary Engineering Research Laboratory, University of California, Berkeley, SERL Report No. 67-7, May 1967.

Gomory, R. E., Outline of an algorithm for integer solutions to linear programs, Bulletin of the American Mathematical Society 64, 275-278, 1958.

Hausman, W. H., and Peter Gilmour, A multi-period truck delivery problem, Transportation Research, Volume 1, 349-357, Pergamon Press, 1967.

Hayes, R. L., The delivery problem, doctoral dissertation, Carnegie Institute of Technology, 1967.

Held, M., and R. M. Karp, A dynamic programming approach to sequencing problems, Journal of the Society for Industrial and Applied Mathematics, 10, 196-210, 1962.

Hess, S. W., and Weaver, J. B., Nonpartisan political redistricti by computer, Operations Research, 13, 998, 1965.

Kuehn, A. A., and M. J. Hamburger, A heuristic program for locati warehouses, Management Science, 9, 643-666, 1963.

Land, A. H., and A. G. Doig, An automatic method of solving discr programming problems, Econometrics, 28, 497-520, 1960.

Liebman, J. C., and D. H. Marks, A Balas algorithm for the zoned uniform treatment problem, Journal of the Sanitary Engineering Division, American Society of Civil Engineers, 94, SA4, August

Little, J. D. C., K. G. Murty, D. W. Sweeney, and C. Karel, An algorithm for the traveling salesman problems. Operations Research, 11, 972-989, 1963.

Ludwig, H. F., and R. J. Black, Report on the solid waste problem Journal of the Sanitary Engineering Division, American Society Civil Engineers, 94, SA2, April 1968.

Mills, G., A decomposition algorithm for the shortest-route probl Operations Research Society of America, 14, 279-291, 1966.

Pierce, J. F., On the truck dispatching problem - Part 1, IBM Cambridge Scientific Center Report 320-2018, Cambriige, Massach setts, July 1967. Revised December 1967.

. F., Applicstion of combinational programming to a class
zero-one integer programming problems, Management Science,
, November 1968.

. F., Private communication, February 1969.

. F., and D. J. Hatfield, Production sequencing by combi-
l programming, IBM Cambridge Scientific Center Report 320-
ambridge, Massachusetts, 1966.

. F., and J. S. Lasky, An improved combinational programming
hm for zero-one integer programming, to be presented at the
of the Operations Research Society of Americs, June 17,
enver, Colorado, as paper III D.5. Abstract in Bulletin of
rations Research Society of America, 17, Sup. 1, Spring 1969.

E., A. Charnes, and S. J. Wersan, Simulation and analysis
fuse collection system, Journal of the Sanitary Engineering
n, American Society of Civil Engineers, 91, SA5, October

E., M. Tanaka, and A. Charnes, Refuse quantities and the
cy of service, Journal of the Sanitary Engineering Division,
n Society of Civil Engineers, 94, SA2, April 1968.

E., M. Tanaka, and S. J. Wersan, Simulation model of refuse
ion policies, Journal of the Sanitary Engineering Division,
n Society of Civil Engineers, 95, SA3, June 1969.

L., National Bureau of Standards, Gaithersburg, Maryland,
communication, July 1968.

ichael J., Planning for regional refuse systems, Ph.D. thesis,
University, September 1968.

, K., An algorithm for the simple plant location problem
me side conditions, Operations Research, 17, January-
y 1969.

Murray, The computer art of schedule making, Datamation,
April 1969.

, The truck assignment problem, Naval Logistics Review
ly, 529-557, 1967.

M., J. C. Liebman, and C. W. Kruse, An investigation of
aste policies, Department of Environmental Engineering
The Johns Hopkins University, Baltimore, Maryland,
1968.

Truitt, M. M., J. C. Liebman, and C. W. Kruse, Simulation mode urban refuse collection, Journal of the Sanitary Engineering Division, American Society of Civil Engineers, 95, SA2, Apri

Walker, W., Adjacent extreme point algorithm for the fixed cha problem, Center for Environmental Quality Management and Department of Operations Research, Cornell University, Ithac New York, January 30, 1968.

White, W. W., Dynamic transshipment networks: an algorithm and application to the distribution of empty containers, IBM New Scientific Center, Technical Report No. 320-2966, February 1

Wolfe, H., and R. Zinn, Systems analysis of solid waste dispos problems, Public Works, 98, September 1967.

Zandi, I., and J. Hayden, Collection of municipal solid wastes pipelines, American Society of Civil Engineers National Meet of Transportation Engineering, Preprint #640, San Diego, California, February 1968.